British Journal of Mathematics & Computer Science
17(1): 1-5, 2016, Article no.BJMCS.26315
ISSN: 2231-0851

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A New Viewpoint in the Study of Quasi b-components in Bitopological Spaces

A. A. Nasef[1*] and A. A. Azzam[2]

[1] *Department of Physics and Engineering Mathematics, Faculty of Engineering, Kafr El-Sheikh University, Kafr El-Sheikh, Egypt.*
[2] *Department of Mathematics, Faculty of Science, Assuit University, New Valley, Egypt.*

Authors' contributions

This work was carried out in collaboration between two authors. Author AAN designed the study, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Author AAA managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/26315
Editor(s):
(1) Feyzi Basar, Department of Mathematics, Fatih University, Turkey.
Reviewers:
(1) Fernando de Lima Caneppele, University of So Paulo, Brazil.
(2) A. S. Salama, Shaqra University, Saudi Arabia.
Complete Peer review History: http://sciencedomain.org/review-history/14795

Received: ***11th April 2016***
Accepted: ***17th May 2016***
Published: ***27th May 2016***

Short Research Article

Abstract

In this paper we introduce and study quasi b-components in bitopological spaces using b-open sets due to Andrijevic in 1996. This concept is obtained by generalizing the idea of quasi-components due to Reilly and Young in 1974.

Keywords: Topological spaces; b-open sets; b-components.

AMS Mathematics (2000) Math. Subject Classification: Primary: 54B05,54C08; Secondary: 54D05.

*Corresponding author: E-mail: nasefa50@yahoo.com;

1 Introduction and Preliminaries

In 1963, Kelly [1] defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies $\tau_1$and τ_2 on X and he initiated a systematic study of bitopological spaces. After the works of Kelly on bitopological space, various authors (e.g. [2, 3, 4, 5, 6]) turned their attention to the generalization of various concepts of topology by considering bitopological spaces instead of topological spaces. In 2003, the first author and Abo Khadra [7] extended the concept of b-open sets and b-closed sets to the setting of bitopological spaces. In the present paper we define and study quasi-b-components in bitopological spaces. This is obtained by generalizing the concept of quasi-component in bitopological spaces due to Reilly and Young [8]. For a subset A of X, τ_i-$Cl(A)$ (resp. τ_i-$Int(A)$) denotes the closure (resp. interior) of A with respect to τ_i for $i = 1, 2$. However, τ_i-$Cl(A)$ and τ_i-$Int(A)$ are briefly denoted by $Cl_i(A)$ and $Int_i(A)$, respectively, if there is no possibility of confusion. This paper is closely related to [9].

Definition 1.1. (see[7]). A subset A of a bitopological space (X, τ_1, τ_2) is said to be:

(a) (τ_i, τ_j)- b-open (briefly (i, j)-b-open) if $A \subset Int_i(Cl_j(A)) \cup Cl_j(Int_i(A))$;
(b) (τ_i, τ_j)- b-closed (briefly (i, j)-b-closed) if its complement is (i, j)-b-open, equivalently, $A \supset Cl_j(Int_i(A)) \cap Int_i(Cl_j(A))$ where $i \neq j, i, j = 1, 2$.

Definition 1.2. (see[7]). Let A be a subset of a bitopological space (X, τ_1, τ_2). Then

(a) The intersection of all (i, j)-b-closed sets of (X, τ_1, τ_2) containing A is called the (τ_i, τ_j)-b-closure of A and is denoted by (τ_i, τ_j)-$bCl(A)$ (briefly (i, j)-$bCl(A)$).

(b)The union of all (i, j)-b-open sets of (X, τ_1, τ_2) contained in A is called the (τ_i, τ_j)-b-interior of A and is denoted by $(\tau_i, \tau_j) - bInt(A)$ (briefly $(i, j) - bInt(A)$) where $i \neq j, i, j = 1, 2$.

The collection of all (i, j)-b-open sets of a bitopological space (X, τ_1, τ_2) will be denoted by (i, j)-BO(X) for $i \neq j$ and i, j = 1, 2. A subset A of a bitopological space (X, τ_1, τ_2) is said to be pairwise b-open if it is (1, 2)-b-open and (2, 1)-b-open. Also A is said to be pairwise b-closed if it is (1, 2)-b-closed and (2, 1)-b-closed.

2 Quasi-b-components in Bitopological Spaces

In this section we define and study pairwise b-component, pairwise totaly b-disconnected and τ_1-locally b-connected in bitopological space (X, τ_1, τ_2).

Definition 2.1. A bitopological space (X, τ_1, τ_2) is said to be pairwise b-connected if X cannot be expressed as the union of two non-empty pairwise b-separated sets, where A, $B \subset X$ are called pairwise b-separated if $A \cap \tau_1 \setminus bCl(B) = \phi = B \cap \tau_2 \setminus bCl(A)$.

Remark 2.1. If X can be expressed as the union of two non-empty pairwise b-separated sets A, B. Then we write $X = A/B$ and call this a pairwise b-separation of X.

Definition 2.2. Let (X, τ_1, τ_2) be any bitopological space. Define a relation R by $(x, y) \in R$ if and only if x and y cannot be separated by a pairwise b-separation A/B of X, where x, $y \in X$. This is equivalent to saying that for each pairwise b-separation A/B of X either $x \in A$ and $y \in A$ or $x \in B$ and $y \in B$. It is easy to see that R is an equivalence relation.

Definition 2.3. Let x be any point of a bitopological space $(X, \tau_1, \tau_2$). The equivalence class of x with respect to R is said to be the quasi-b-component of x and is denoted by $(QB)_x$.

Remark 2.2. A quasi-component of a bitopological space (X, τ_1, τ_2) need not be a quasi semi-component [9] and consequently need not be a quasi b-component as shown as by the following example.

Example 2.1. *Let $X = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{b\}, \{b, c\}\}$ and $\tau_2 = \{X, \phi, \{c\}, \{b, c\}\}$. Let $A = \{b\}$ and $B = \{a, c\}$. Then $X = A/B$, which is a pairwise b-separation of X. Here $b \in A$ and $c \in B$. Thus b, c do not belong to a quasi b-component of X. But we see that b, c belong to a quasi-component of X, since the points b and c are not (pairwise) separated.*

Theorem 2.1. *A quasi-b-component of a bitopological space (X, τ_1, τ_2) is contained in a quasi-component.*

Proof. The proof is obvious in view of the fact that any two pairwise separated sets in (X, τ_1, τ_2) are pairwise b-separated. □

Remark 2.3. Reilly and Young [7] have shown that quasi-components of a bitopological space (X, τ_1, τ_2) are not related to the quasi-components of the spaces (X, τ_1) and (X, τ_2). Here we take the same example to show that quasi b-components of (X, τ_1, τ_2) are not related to quasi b-components of (X, τ_1) and (X, τ_2). The example was: $X = \{a, b\}$, $\tau_1 = \{X, \phi, \{a\}\}$ and $\tau_2 = \{X, \phi, \{b\}\}$. Clearly $\{a\}/\{b\}$ is a pairwise b-separation of X and so $(QB)_a = \{a\}$. But quasi-b-component of a in both (X, τ_1) and (X, τ_2) is X.

Definition 2.4. A subset Y of a bitopological space $(X, \tau_1, \tau_2)$is said to be pairwise b-connected if Y cannot be expressed as the union of two non-empty pairwise b-separated sets in X.

Definition 2.5. Let (X, τ_1, τ_2)be any bitopological space. Then a maximal pairwise b-connected set in X is called a b-component of X and if x belongs to this b-component say $(B.C.)$, then it will be denoted by $B.C.(x)$.

Theorem 2.2. *In a bitopological space (X, τ_1, τ_2),every quasi b-component is the union of the b-components of its points.*

Proof. Let $(QB)_x$ be a quasi b-component of X containing x. We are to show that $(QB)_x = \cup\{B.C.(q) : q \in (QB)_x\}$, where, $B.C.(q)$ denote b-components in (X, τ_1, τ_2). Let $y \in B.C.(q)$. Then $(y, q) \in R$. But $q \in (QB)_x$, therefore $(q, x) \in R$ and so $(y, x) \in R$. This implies that $y \in (QB)_x$. Hence, $B.C.(q) \subset (QB)_x$ for each $q \in (QB)_x$. This means that$\cup[B.C.(q) : q \in (QB)_x] \subset (QB)_x$. The inclusion$(QB)_x \subset \cup[B.C.(q) : q \in (QB)_x]$ is clear. □

Theorem 2.3. *The following statements are true for any bitopological space (X, τ_1, τ_2).*
(i) Every b-component is contained in a quasi b-component.
(ii) Every quasi b-component is a union of b-components.
(iii) A quasi-b-component is a b-component if and only if it is pairwise b-connected.

Proof. Follows directly from Theorem 2.11. □

Definition 2.6. A bitopological space $(X, \tau_1, \tau_2)$is said to be pairwise totally b-disconnected if each pair of points of X can be separated by a pairwise b-separation of X.

Theorem 2.4. *A bitopological space $(X, \tau_1, \tau_2)$is pairwise totally b-disconnected if and only if quasi-b-components of X are singletons.*

Proof. Obvious. □

Definition 2.7. (EL-Atik [10]) A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be b-irresolute if the inverse image of every b-open set is b-open.

Theorem 2.5. *Let $f : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ be a pairwise b-irresolute pairwise open surjection. Then the image of a quasi b-component of X lies in a quasi b-component of Y.*

Proof. The proof is obvious in view of Theorem 2.11. □

Definition 2.8. Let (X, τ_1, τ_2) be a bitopological space. Then X is said to be τ_1-locally b-connected with respect to τ_2 if for each $x \in X$ and every τ_1-b-open set U containing x, there exists a pairwise b-connected τ_1-open set G such that $x \in G \subset U$. A bitopological space (X, τ_1, τ_2) is said to be pairwise locally b-connected if it is τ_1-locally b-connected with respect to τ_2 and τ_2-locally b-connected with respect to τ_1.

Theorem 2.6. *If a bitopological space (X, τ_1, τ_2) is pairwise locally b-connected, then the b-components of X are bi-b-closed.*

Theorem 2.7. *In a pairwise locally b-connected space (X, τ_1, τ_2) ,each quasi b-component is a b-component.*

Proof. Let (X, τ_1, τ_2) be a pairwise locally b-connected space and let $x \in X$. Then with the usual notations of quasi b-components of X, we have $(QB)_x = \cup\{B.C.(g) : g \in (QB)_x\}$. Clearly, $B.C.(x) \subset (QB)_x$. We claim that $(QB)_x = B.C.(x)$. Let $y \in (QB)_x \setminus B.C.(x)$. Since $B.C.(x)$ is τ_1-open and τ_1-b-closed (Theorem 2.18), therefore, $X = B.C.(x)/[X \setminus B.C.(x)]$ is a pairwise b-separation of X with $x \in B.C.(x)$ and $y \in X \setminus B.C.(x)$. Thus $y \notin (QB)_x$, which is a contradiction. Hence $(QB)_x = B.C.(x)$. □

Theorem 2.8. *If x is any point of a bitopological space $(X, \tau_1, \tau_2), \{W_a : a \in A\}$ is the family of all τ_1-b-open τ_2-b-closed sets containing x, and $\{V_b : b \in B\}$ is the family of all τ_1-b-closed τ_2-b-open sets containing x, then $(QB)_x = (\bigcap_a W_a) \bigcap (\bigcap_b V_b)$.*

Proof. Follows directly from the above definitions. □

Corollary 2.1. *Any quasi-b-component $(QB)_x$ of a bitopological space (X, τ_1, τ_2) satisfies the relation: $(QB)_x = \tau_1 - bCl[(QB)_x] \cap \tau_2 - bCl[(QB)_x]$.*

Proof. With the notations of Theorem 2.20, $\bigcap_b V_b$ is a $\tau_1 - b-$closed set containing $(QB)_x$, so that $\tau_1 - bCl[(QB)_x] \subset \bigcap_b V_b$. Similarly $\tau_2 - bCl[(QB)_x] \subset \bigcap_a W_a$. Hence $\tau_1 - bCl[(QB)_x] \bigcap \tau_2 - [(QB)_x] \subset (\bigcap_a W_a) \bigcap (\bigcap_b V_b) = (QB)_x$. Obviously, $(QB)_x \subset \tau_1 - bCl[(QB)_x] \bigcap \tau_2 - bCl[(QB)_x]$ and so the relation is satisfied. □

3 Conclusion

Some bitopological concepts and their characterizations are obtained. Therefore the results of Arya and Nour [9] are improvements.

Acknowledgements

The authors are grateful to referees for valuable comments and suggestions in improving this paper.

Competing Interests

Authors have declared that no competing interests exist.

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