British Journal of Mathematics & Computer Science
11(6): 1-13, 2015, Article no.BJMCS.20552
ISSN: 2231-0851

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Algorithms and Some New Traveling Wave Solutions via New Extension of (G'/G)-expansion Method of Coupled Nonlinear Equations

Hasibun Naher[1*]

[1]*Department of Mathematics and Natural Sciences, BRAC University, 66 Mohakhali, Dhaka 1212, Bangladesh.*

Article Information

DOI: 10.9734/BJMCS/2015/20552
Editor(s):
(1) Sergio Serrano, Department of Applied Mathematics, University of Zaragoza, Spain.
Reviewers:
(1) Anonymous, Mansoura University, Egypt.
(2) Anonymous, Pabna University of Science and Technology, Bangladesh.
(3) Anonymous, Moscow Lomonosov State University, Russia.
Complete Peer review History: http://sciencedomain.org/review-history/11453

Original Research Article

Received: 31 July 2015
Accepted: 27 August 2015
Published: 17 September 2015

Abstract

In this work, the second order nonlinear ordinary differential equation is implemented as an auxiliary equation. For illustration, the generalized Hirota-Satsuma coupled KdV equations are considered for constructing traveling wave solutions by applying a new extension of so called (G'/G) method. As a result, many new traveling wave solutions have been generated with many arbitrary parameters. The obtained solutions also show the wider applicability of this new extended method for handling nonlinear evolution equations. The numerical results are also described in the figures.

Keywords: *New extension of (G'/G)-expansion method; exact solutions; the hirota-satsuma coupled KdV equations; auxiliary equation; nonlinear ordinary differential equation; traveling wave solutions.*

1 Introduction

The study of coupled nonlinear partial differential equations (PDEs) is one of the main themes in nonlinear science. Due to important applications of nonlinear evolution equations (NLEEs) in real world problems, it is required to construct new analytical solutions. In particular, the construction of analytical solutions for coupled nonlinear equations play an important role in knowing facts that are not simply understand by

*Corresponding author: E-mail: hasibun06tasauf@gmail.com;

common observations. In the past several decades, a variety of powerful methods have been introduced, such as the Bäcklund transformation method [1], the Hirota's bilinear method [2], the inverse scattering method [3], the Jacobi elliptic function method [4,5], the tanh-coth method [6], the F-expansion method [7], the exp-function method [8,9] the modified simple equation method [10,11], the homogeneous balance method [12], Adomian decomposition method [13,14] and so on.

Recently, Wang et al. [15] introduced basic $\left(G'/G\right)$ -expansion method and has become widely used to generate exact solutions of NLEEs. This method is based on second order linear ordinary differential equation (ODE). Afterwards, a group of scientists applied this method to investigate various nonlinear PDEs for constructing traveling wave solutions. Bekir [16] applied this method to the Boussinesq equation, the modified Zakharov-Kuznetsov equation and the Konopelchenko-Dubrovsky equations in the same research paper. Aslan [17] constructed the analytical solutions by applying this method to the modified Degasperis-Procesi equation, the Burgers-KdV equation and the modified Benjamin-Bona-Mahony equations while Zayed [18] studied higher dimensional some nonlinear evolution equations, such as the (3+1)-dimensional YTSF equation, the (3+1)-dimensional shallow water equation, the (3+1)-dimensional Kadomtsev-Petviashvili equation, the (3+1)-dimensional KdV-Zakharov-Kuznetsov equation and the (3+1)-diemsional Jimbo-Miwa equation via the same method to establish exact solutions. Naher et al. [19] constructed abundant traveling wave solutions of the higher-order Caudrey-Dodd-Gibon equation via this powerful method.

In order to show the effectiveness of the basic $\left(G'/G\right)$ -expansion method and the use of wider applicability, further research is carried out by a rich class of scientists. For instance, Zhang et al. [20] extended the $\left(G'/G\right)$ method as the improved $\left(G'/G\right)$ -expansion method. Subsequently, many researchers studied various nonlinear PDEs to construct traveling wave solutions [21-27].

Moreover, Akbar et al. [28] introduced the generalized and improved $\left(G'/G\right)$ -expansion method. Consequently, Naher et al. [29] implemented this method to construct traveling wave solutions of the (3+1)-dimensional nonlinear PDE. Furthermore, Khan et al. [30,31] studied some nonlinear PDEs via the enhanced $\left(G'/G\right)$ -expansion method.

The present work is based on the nonlinear ODE as an auxiliary equation with many arbitrary parameters to produce many new traveling wave solutions. To illustrate the power of this new extended method, we apply to the generalized Hirota-Satsuma coupled KdV equations.

2 Description of the Method

The general nonlinear PDE is considered as below:

$$Q\left(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots\right) = 0, \tag{1}$$

where $u = u\left(x,t\right)$ is an unknown function, Q is a polynomial in $u\left(x,t\right)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved.

The algorithms of new extension with nonlinear ODE as auxiliary equation is as follows:

Step 1: Suppose that the combination of variables x and t by a new variable ξ

$$u(x,t)=u(\xi), \quad \xi = x \pm Vt, \tag{2}$$

where V is the speed of the travelling wave. Now using travelling wave transformation Eq. (2), Eq. (1) is transformed into ODE for $u=u(\xi)$:

$$F(u,u',u'',u''',\ldots)=0, \tag{3}$$

where F is a function of $u(\xi)$ and its total derivatives.

Step 2: Integrating Eq. (3) (whenever possible). For simplicity, the integral constant may be zero.

Step 3. Suppose that the travelling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi)=\sum_{j=0}^{m} a_j \left(d+G'/G\right)^j, \tag{4}$$

where $a_m \neq 0$, and $G=G(\xi)$ satisfies new second order nonlinear ODE:

$$GG''=AG^2+BGG'+C(G')^2, \tag{5}$$

where prime denotes the derivative with respect to ξ; A, B and C are real parameters.

Step 4. The positive integer m can be determined as considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 5. Substituting Eq. (4) along with Eq. (5) into Eq. (3) with the value of m which obtained in step 4, we obtain polynomials in $\left(d+G'(\xi)/G(\xi)\right)^m$ $(m=0,1,2,\ldots)$. Then collecting each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for $a_j\,(j=0,1,2,\ldots,m)$, d, A, B, C and V.

Step 6. Solving the system of algebraic equations which were obtained in step 5. Suppose that the value of constants $a_j\,(j=0,1,2,\ldots,m), d$ and V can be obtained by solving the algebraic equations obtained in step 5. Then, substituting the values of constants together with general solutions of Eq. (5) into Eq. (4), we obtain new travelling wave solutions of the nonlinear Eq. (1).

The difference between basic (G'/G) method and this new extension of the (G'/G) method with a particular nonlinear ODE (Eq. 5) is used as an auxiliary equation instead of linear ODE.

It is significant to point out that the solutions of Eq. (5) for (G'/G) in the forms of the hyperbolic function, the trigonometric function and the rational forms as given below:

Family 1: Hyperbolic Function Solutions, when $B \neq 0,\ \Omega = B^2+4A(1-C)>0,\ \Psi = 1-C$,

$$\left(\frac{G'}{G}\right)=\frac{B}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)},\tag{6}$$

where C_1 and C_2 are arbitrary constants.

When and $\Delta=\Psi A>0,$
$B=0,\Psi=1-C$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)},\tag{7}$$

where C_1 and C_2 are arbitrary constants.

Family 2. Trigonometric Function Solutions, when $B\neq 0,\ \Omega=B^2+4A(1-C)<0,$

$$\Psi=1-C\,,\quad \left(\frac{G'}{G}\right)=\frac{B}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)},\tag{8}$$

where C_1 and C_2 are arbitrary constants.

When $B=0,\Psi=1-C$ and $\Delta=\Psi A<0,$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)},\tag{9}$$

where C_1 and C_2 are arbitrary constants.

Family 3. Rational Form Solution, when $B\neq 0,\ \Omega=B^2+4A(1-C)=0,\ \Psi=1-C$

$$\left(\frac{G'}{G}\right)=\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi},\tag{10}$$

where C_1 and C_2 are arbitrary constants.

3 Application of the Method

In this section, we propose to illustrate new extension of the (G'/G) method by implementing it to the Hirota-Satsuma coupled KdV equations.

The Hirota-Satsuma coupled KdV equations are [32]:

$$u_t - \frac{1}{2}u_{xxx} + 3uu_x - 3(vw)_x = 0, \tag{11}$$

$$v_t + v_{xxx} - 3uv_x = 0, \tag{12}$$

$$w_t + w_{xxx} - 3uw_x = 0. \tag{13}$$

Now using the travelling wave transformation Eq. (2) into Eq. (11) to Eq. (13), yield:

$$-V\,u' - \frac{1}{2}u''' + 3uu' - 3(vw)' = 0, \tag{14}$$

$$-V\,v' + v''' - 3uv' = 0, \tag{15}$$

$$-V\,w' + w''' - 3uw' = 0. \tag{16}$$

Taking the homogeneous balance between uu' and u''' in Eq. (14); uv' and v''' in Eq. (15); and uw' and w''' in Eq. (16); yield $m_1 = 2$, $m_2 = 2$ and $m_3 = 2$. Thus the solutions take the form:

$$u = a_0 + a_1\left(d + (G'/G)\right) + a_2\left(d + (G'/G)\right)^2, \tag{17}$$

$$v = b_0 + b_1\left(d + (G'/G)\right) + b_2\left(d + (G'/G)\right)^2, \tag{18}$$

$$w = c_0 + c_1\left(d + (G'/G)\right) + c_2\left(d + (G'/G)\right)^2, \tag{19}$$

where $a_2, a_1, a_0, b_2, b_1, b_0, c_2, c_1, c_0$ and d are arbitrary constants to be determined. Substituting Eq. (17) to Eq. (19) together with Eq. (5) into Eq. (14) to Eq. (16), the left-hand side is converted into polynomials in $\left(d + G'(\xi)/G(\xi)\right)^m$ $(m = 0,1,2,...)$. Collecting the coefficients of like power of these polynomials to zero; yield a set of algebraic equations for $a_2, a_1, a_0, b_2, b_1, b_0, c_2, c_1, c_0, d, A, B, C$ and V . The above systems of algebraic equations have been solved and yields one set of solutions with many parameters.

Set 1:

$$\begin{aligned}&a_2=4\Psi^2, a_1=-4\Psi\left(B+2d\Psi\right), b_1=\frac{4\Psi^2}{c_2}\left(8dC-B\Psi-2d\right), b_2=\frac{4\Psi^4}{c_2},\\&c_1=\frac{-c_2}{\Psi}\left(B+2d\Psi\right), d=d,\\&V=\frac{1}{8c_2\Psi^2}\begin{Bmatrix}24c_2d^2\Psi^4-16c_2A\Psi^2+24c_2dB\Psi^2-24c_2dBC\Psi+2c_2B^2\Psi^2\\+12c_0C\Psi^3+12c_0C^3+36c_0C\Psi-3b_0c_2^2-12c_0\end{Bmatrix},\end{aligned}\tag{20}$$

where $\Psi=1-C, a_0, b_0, c_2, c_0, d, A, B$ and C are free parameters.

Hyperbolic Function Solutions: When $B\neq 0,\ \Omega=B^2+4A(1-C)>0,\ \Psi=1-C,$ substituting Eq. (20) together with Eq. (6) into Eq. (17) to Eq. (19) and after simplifying the solutions become (if $C_1=0$ but $C_2\neq 0$) respectively:

$$u_1\left(x,t\right)=a_0-\left\{B^2-\Omega\coth^2\left(\left(\sqrt{\Omega}/2\Psi\right)\xi\right)+4d\Psi\left(B+d\Psi\right)\right\},$$

$$v_1\left(x,t\right)=b_0+b_1\left\{d+\frac{1}{2\Psi}\left(B+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)\right\}+b_2\left\{d+\frac{1}{2\Psi}\left(B+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)\right\}^2$$

$$w_1\left(x,t\right)=c_0-c_2\left\{\frac{1}{4\Psi^2}\left(B^2-\Omega\coth^2\left(\left(\sqrt{\Omega}/2\Psi\right)\xi\right)\right)+d\left(\frac{B}{\Psi}+d\right)\right\},$$

where $\xi=x-\dfrac{1}{8c_2\Psi^2}\begin{Bmatrix}24c_2d^2\Psi^4-16c_2A\Psi^2+24c_2dB\Psi^2-24c_2dBC\Psi+2c_2B^2\Psi^2\\+12c_0C\Psi^3+12c_0C^3+36c_0C\Psi-3b_0c_2^2-12c_0\end{Bmatrix}t$.

Moreover, substituting Eq. (20) together with Eq. (6) into Eq. (17) to Eq. (19) and simplifying, the travelling wave solutions become (if $C_2=0$ but $C_1\neq 0$) respectively:

$$u_2\left(x,t\right)=a_0-\left\{B^2-\Omega\tanh^2\left(\left(\sqrt{\Omega}/2\Psi\right)\xi\right)+4d\Psi\left(B+d\Psi\right)\right\},$$

$$v_2\left(x,t\right)=b_0+b_1\left\{d+\frac{1}{2\Psi}\left(B+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)\right\}+b_2\left\{d+\frac{1}{2\Psi}\left(B+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)\right\}^2$$

$$w_2\left(x,t\right)=c_0-\frac{c_2}{\left(2\Psi\right)^2}\left\{B^2-\Omega\tanh^2\left(\left(\sqrt{\Omega}/2\Psi\right)\xi\right)+4d\Psi\left(B+d\Psi\right)\right\},$$

When $B=0, \Psi=1-C, \Delta=\Psi A>0,$ substituting Eq. (20) together with Eq. (7) into Eq. (17) to Eq. (19) and simplifying, the generated solutions become (if $C_1=0$ but $C_2\neq 0$) respectively:

$$u_3(x,t) = a_0 - 4\left\{\sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)\left(B - \sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)\right) + d\Psi\left(B + d\Psi\right)\right\},$$

$$v_3(x,t) = b_0 + b_1\left\{d + \frac{\sqrt{\Delta}}{\Psi}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\} + b_2\left\{d + \frac{\sqrt{\Delta}}{\Psi}\coth\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2,$$

$$w_3(x,t) = c_0 - c_2\left\{\frac{\sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)}{\Psi^2}\left(B - \sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)\right) + d\left(\frac{B}{\Psi} + d\right)\right\}.$$

Furthermore, substituting Eq. (20) together with Eq. (17) into Eq. (17) to Eq. (19) and simplifying, the generated solutions become (if $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_4(x,t) = a_0 - 4d\Psi\left(B + d\Psi\right) - 4\left\{B\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right) + \Delta\left(1 - \operatorname{sech}^2\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)\right)\right\},$$

$$v_4(x,t) = b_0 + b_1\left\{d + \frac{\sqrt{\Delta}}{\Psi}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\} + b_2\left\{d + \frac{\sqrt{\Delta}}{\Psi}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2,$$

$$w_4(x,t) = c_0 - \frac{dc_2}{\Psi}\left(B + d\Psi\right) - \frac{c_2}{\Psi^2}\left\{B\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right) - \Delta\left(1 - \operatorname{sech}^2\left(\left(\sqrt{\Delta}/\Psi\right)\xi\right)\right)\right\}.$$

Trigonometric Function Solutions: When $B \neq 0, \Omega = B^2 + 4A(1-C) < 0, \Psi = 1 - C,$ substituting Eq. (20) together with Eq. (8) into Eq. (17) to Eq. (19) and simplifying, the solutions become (if $C_1 = 0$ but $C_2 \neq 0$) respectively:

$$u_5(x,t) = a_0 - \left\{B^2 + \Omega\cot^2\left(\left(\sqrt{-\Omega}/2\Psi\right)\xi\right) + 4d\Psi\left(B + d\Psi\right)\right\},$$

$$v_5(x,t) = b_0 + b_1\left\{d + \frac{1}{2\Psi}\left(B + i\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)\right\} + b_2\left\{d + \frac{1}{2\Psi}\left(B + i\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)\right\}^2,$$

$$w_5(x,t) = c_0 - \frac{c_2}{4\Psi^2}\left\{B^2 + \Omega\left(\csc^2\left(\left(\sqrt{-\Omega}/2\Psi\right)\xi\right) - 1\right)\right\} - c_2 d\left(\frac{B}{\Psi} + d\right).$$

Again, substituting Eq. (20) together with Eq. (8) into Eq. (17) to Eq. (19) and simplifying, the travelling wave solutions become (if $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_6(x,t) = a_0 - \left\{B^2 + \Omega\left(\sec^2\left(\left(\sqrt{-\Omega}/2\Psi\right)\xi\right) - 1\right)\right\} - 4d\Psi\left(B + d\Psi\right),$$

$$v_6(x,t)=b_0+b_1\left\{d+\frac{1}{2\Psi}\left(B+i\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)\right\}+b_2\left\{d+\frac{1}{2\Psi}\left(B+i\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right)\right\}^2,$$

$$w_6(x,t)=c_0-\frac{c_2}{4\Psi^2}\left\{B^2+\Omega\tan^2\left(\left(\sqrt{-\Omega}/2\Psi\right)\xi\right)\right\}-c_2d\left(\frac{B}{\Psi}+d\right).$$

When $B=0,\Psi=1-C,\Delta=\Psi A<0,$ substituting Eq. (20) together with Eq. (9) into Eq. (17) to Eq. (19) and simplifying, the solutions become (if $C_1=0$ but $C_2\neq 0$) respectively:

$$u_7(x,t)=a_0-4\left\{iB\sqrt{\Delta}\cot\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)-\Delta\cot^2\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)+d\Psi\left(B+d\Psi\right)\right\},$$

$$v_7(x,t)=b_0+b_1\left\{d+\frac{i\sqrt{\Delta}}{\Psi}\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}+b_2\left\{d+\frac{i\sqrt{\Delta}}{\Psi}\cot\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2,$$

$$w_7(x,t)=c_0-\frac{c_2d\left(B+d\Psi\right)}{\Psi}-\frac{ic_2\sqrt{\Delta}}{\Psi^2}\left\{B\cot\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)-i\sqrt{\Delta}\cot^2\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)\right\}.$$

Further, substituting Eq. (20) together with Eq. (9) into Eq. (17) to Eq. (19) and simplifying, the solutions become (if $C_2=0$ but $C_1\neq 0$) respectively:

$$u_8(x,t)=a_0+4i\sqrt{\Delta}\left\{B\frac{\sin\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)}{\cos\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)}+i\sqrt{\Delta}\left(\sec^2\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)-1\right)\right\}-4d\Psi\left(B+d\Psi\right),$$

$$v_8(x,t)=b_0+b_1\left\{d-\frac{i\sqrt{\Delta}}{\Psi}\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}+b_2\left\{d-\frac{i\sqrt{\Delta}}{\Psi}\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2,$$

$$w_8(x,t)=c_0-\frac{c_2d}{\Psi}\left(B+d\Psi\right)+\frac{ic_2\sqrt{\Delta}}{\Psi^2}\left\{\frac{B\sin\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)\cos\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)+i\sqrt{\Delta}\sin^2\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)}{\cos^2\left(\left(\sqrt{-\Delta}/\Psi\right)\xi\right)}\right\}.$$

Rational Form Solutions: When $B\neq 0,\Omega=B^2+4A\left(1-C\right)=0,\Psi=1-C,$ substituting Eq. (20) together with Eq. (10) into Eq. (17) to Eq. (19) and simplifying, the solutions become respectively:

$$u_9(x,t)=a_0-4d\Psi\left(B+d\Psi\right)-\left\{B^2-\left(\frac{2C_2\Psi}{C_1+C_2\xi}\right)^2\right\},$$

$$v_9(x,t)=b_0+b_1\left(d+\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)+b_2\left(d+\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)^2,$$

$$w_9(x,t)=c_0-\frac{c_2 d}{\Psi}(B+d\Psi)-c_2\left\{\left(\frac{B}{2\Psi}\right)^2-\left(\frac{2C_2\Psi}{C_1+C_2\xi}\right)^2\right\},$$

where C_1 and C_2 are arbitrary constants and

$$\xi=x-\frac{24c_2d^2\Psi^4-16c_2A\Psi^2+24c_2dB\Psi^2-24c_2dBC\Psi+2c_2B^2\Psi^2+12c_0C\Psi^3+12c_0C^3+36c_0C\Psi-3b_0c_2^2-12c_0}{8c_2\Psi^2}t.$$

4 Results and Discussion

4.1 Comparisons

Many researchers studied the Hirota-Satsuma coupled KdV equations by using various approaches. For example, Yu et al. [33] solved these equations via the Jacobi elliptic function method. Abbasbandy [34] implemted the homotopy analysis method to investigate the same equations whilst Zuo and Zhang [32] studied these equations by applying the basic (G'/G)-expansion method. Good agreement has been noticed between presently generated solutions and published results in the open literature. The newly constructed solutions are compared with the solutions of Aslan [17]; Zuo and Zhang [32]; and Aslan and Özis [35] as below:

(i) If A and B are replaced by $(-\mu)$ and $(-\lambda)$ respectively and $C=0$ in Eq. (5), the nonlinear ODE coincides with the linear ODE (4) of Aslan [17],

(ii) If $d=0$, the Eqs. (17), (18) and (19) identical with Eq. (9) of Aslan [17],

(iii) If A takes $(-\mu)$ and B takes $(-\lambda)$ and $C=0$ in Eq. (5), the nonlinear ODE coincide with the linear ODE (4) of Aslan and Özis [35],

(iv) If $d=0$, the Eqs. (17), (18) and (19) identical with Eq. (7) of Aslan and Özis [35],

(v) If A and B are replaced by $(-\mu)$ and $(-\lambda)$ respectively and $C=0$ in Eq. (5), the nonlinear ODE coincides with the linear ODE (3.7) of Zuo and Zhang [32]. The similarities are also found with Zuo and Zhang [32] as follows:

- If $d=0$, the hyperbolic function solutions u_1 and u_2 identical with the solution Eq. (3.36),
- The solutions v_1 and v_2 coincide with the solution Eq. (3.37) when $d=0$,
- If $d=0$, the hyperbolic function solutions w_1 and w_2 identical with the solution Eq. (3.38).

4.2 Numerical Results

Some obtained solutions are visualized in figures as follows:

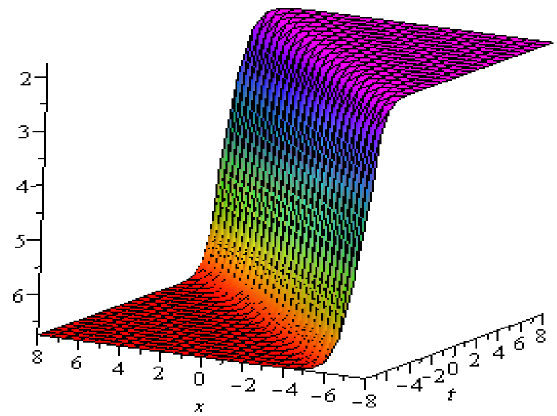

Fig. 1. Kink solution of $v_2(x,t)$ **for** $A=1\times10^{-12}, B=1, C=0.5, c_2=1, c_0=0.1, b_0=1\times10^{-6}, d=1,$ $\xi = x-0.96t$ **with** $-8\le x,\, t\le 8.$

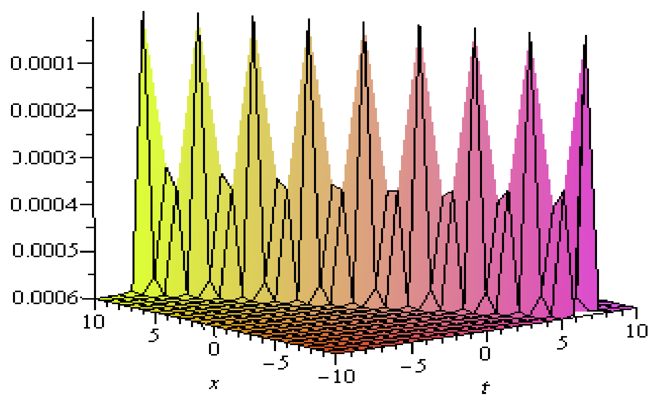

Fig. 2. Solitons of $w_4(x,t)$ **for** $A=0.75, B=0, C=0.5, c_2=1\times10^{-4}, c_0=1\times10^{-6}, b_0=1\times10^{-4},$ $d=1\times10^{-4},\ \xi=x+1.5t$ **with** $-3\le x,\, t\le 3.$

5 Conclusion

A new extension with a particular nonlinear ODE as auxiliary equation has been successfully applied to the generalized Hiroa-Satsuma coupled KdV equations. As a result many new travelling wave solutions are obtained and some of the obtained solutions are in good agreement with those obtained by the basic (G'/G)-expansion method based on some values of parameters. Moreover, the constructed solutions show that the performance of new extension with a particular nonlinear ODE as auxiliary equation is effective and more general and can produce abundant new solutions with many arbitrary parameters.In addition, some of the solutions are illustrated in the figures.

Competing Interests

Author has declared that no competing interests exist.

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