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One-Step Five Offgrid Implicit Hybrid Block Method for the Direct Solution of Stiff Second-Order Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author DR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors EOA and AIM managed the analyses of the study. Author LA managed the literature searches. All authors read and approved the final manuscript.

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Abstract

A one-step five off-grid implicit hybrid block method for the solution of stiff Second Order Ordinary Differential Equations is developed. The continuous hybrid linear multistep method was generated using power series approximation via interpolation and collocation at the grid points. The discrete block method was recovered by evaluating the continuous block method at some selected grid points. The method is convergent. Numerical experiments on some Second Order Ordinary differential equations show that the method is more accurate than some existing methods.

Keywords: One-step; hybrid block; five off-step; stiff ODEs.

1 Introduction

This paper considers second order initial value problems of the form

$$y'' = f(x, y(x), y'(x)),\quad y(x_0) = y_0,\ y'(x_0) = y'_0 \tag{1}$$

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Solving higher order derivatives method by reducing them to a system of first-order approach involves more functions to evaluate which then leads to a computational burden as in [1-3]. The setbacks of this approach had been reported by scholars, among them are Bun and Vasil'yer [4].

The method of collocation and interpolation of the power series approximation to generate continuous linear multistep method has been adopted by many scholars, among them are Fatunla [5], Vigor Aquilar and Ramos [6], Adeniran et al [7], Abdelrahim et al. [8], Mohammad et al. [9] to mention a few. Block method generates independent solution at selected grid points without overlapping. It is less expensive in terms of the number of function evaluation compared to the predictor-corrector method. Moreover, it possesses the properties of Runge Kutta method for being self-starting and does not require starting values. Some of the authors that the proposed block method are: [10,11,12].

In this paper, we developed a one-step hybrid block third derivative method with five off-grids, which is implemented in the block method. The implementation of the method is cheaper than the predictor-corrector method. This method harnesses the properties of hybrid and third derivative this makes it efficient for stiff problems.

2 Derivation of the Method

Consider the power series approximate solution of the form

$$y(x)=\sum_{j=0}^{2s+r-1} a_i\left(\frac{x-x_n}{h}\right)^j \tag{2}$$

where $r = 2$ and $s = 4$ are the numbers of interpolation and collocation points respectively, is considered to be a solution to (1).

Differentiating (2) twice gives

$$y''(x)=\sum_{j=2}^{2s+r-1} \frac{a_j j!}{h^2(j-2)!}\left(\frac{x-x_n}{h}\right)^{j-2}=f(x,y,y'), \tag{3}$$

Substituting (3) into (1) gives

$$f(x,y,y'')=\sum_{j=2}^{2s+r-1} \frac{a_j j!}{h^2(j-2)!}\left(\frac{x-x_n}{h}\right)^{j-2} \tag{4}$$

Collocating (4) at all points $x_{n+s}, s=0\left(\frac{1}{6}\right)1$ and Interpolating Equation (2) at $x_{n+r}, r=0,\frac{1}{3}$, gives a system of non-linear equation of the form

$$AX = U \tag{5}$$

where

$$A=\left[a_0\ ,a_1\ ,a_2\,,a_3\,,a_4\,,a_5\,,a_6\,,a_7\,,a_8\right]^T,$$

$$U=\left[y_n, y_{n+\frac{1}{3}}, f_n, f_{n+\frac{1}{6}}, f_{n+\frac{1}{3}}, f_{n+\frac{1}{2}}, f_{n+\frac{2}{3}}, f_{n+\frac{5}{6}}, f_{n+1}\right]^T,$$

and

$$X=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \frac{1}{81} & \frac{1}{243} & \frac{1}{729} & \frac{1}{2187} & \frac{1}{6561} \\ 0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{h^2} & \frac{1}{h^2} & \frac{1}{3h^2} & \frac{5}{54h^2} & \frac{5}{216h^2} & \frac{7}{1296h^2} & \frac{7}{5832h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{2}{h^2} & \frac{4}{3h^2} & \frac{20}{27h^2} & \frac{10}{27h^2} & \frac{14}{81h^2} & \frac{56}{729h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{3}{h^2} & \frac{3}{h^2} & \frac{5}{4h^2} & \frac{15}{8h^2} & \frac{21}{16h^2} & \frac{7}{8h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{4}{h^2} & \frac{8}{3h^2} & \frac{40}{27h^2} & \frac{20}{27h^2} & \frac{84}{243h^2} & \frac{112}{729h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{5}{h^2} & \frac{5}{3h^2} & \frac{25}{54h^2} & \frac{25}{216h^2} & \frac{35}{1296h^2} & \frac{35}{5832h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{6}{h^2} & \frac{12}{h^2} & \frac{20}{h^2} & \frac{30}{h^2} & \frac{42}{h^2} & \frac{56}{h^2} \end{bmatrix}$$

Solving (5) for $a_i's$ using Gaussian elimination method, gives a continuous hybrid linear multistep method of the form

$$y(x)=\sum_{j=0,\frac{1}{3}}\alpha_j y_{n+j}+h^2\left[\sum_{j=0}^{1}\beta_j f_{n+j}+\beta_k f_{n+k}\right]k=\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{5}{6} \tag{6}$$

Differentiating (6) once yields

$$p'(x)=\frac{1}{h}\sum_{j=0,\frac{1}{3}}\alpha_j y_{n+j}+h\left[\sum_{j=\frac{1}{3},\frac{2}{3}}\beta_j f_{n+j}+\sum_{j=0}^{1}\beta_j f_{n+j}\right] \tag{7}$$

where

$$a_0 = 1-\frac{3(x-x_n)}{h}$$

$$a_{\frac{1}{2}} = \frac{3\,(-x_n+x)}{h}$$

$$\beta_0 = -\frac{1027}{22680}\,(-x_n+x)\,h+\frac{1}{2}\,(-x_n+x)^2-\frac{49}{20}\,\frac{(-x_n+x)^3}{h}+\frac{203}{30}\,\frac{(-x_n+x)^4}{h^2}$$
$$-\frac{441}{40}\,\frac{(-x_n+x)^5}{h^3}+\frac{21}{2}\,\frac{(-x_n+x)^6}{h^4}-\frac{27}{5}\,\frac{(-x_n+x)^7}{h^5}+\frac{81}{70}\,\frac{(-x_n+x)^8}{h^6}$$

$$\beta_{\frac{1}{6}} = -\frac{97}{630}\left(-x_n + x\right)h + \frac{6\left(-x_n + x\right)^3}{h} - \frac{261}{10}\frac{\left(-x_n + x\right)^4}{h^2} + \frac{261}{5}\frac{\left(-x_n + x\right)^5}{h^3} - \frac{279}{5}\frac{\left(-x_n + x\right)^6}{h^4} + \frac{216}{7}\frac{\left(-x_n + x\right)^7}{h^5} - \frac{243}{35}\frac{\left(-x_n + x\right)^8}{h^6}$$

$$\beta_{\frac{1}{3}} = \frac{2}{27}\left(-x_n + x\right)h - \frac{15}{2}\frac{\left(-x_n + x\right)^3}{h} + \frac{351}{8}\frac{\left(-x_n + x\right)^4}{h^2} - \frac{4149}{40}\frac{\left(-x_n + x\right)^5}{h^3} + \frac{1233}{10}\frac{\left(-x_n + x\right)^6}{h^4} - \frac{513}{7}\frac{\left(-x_n + x\right)^7}{h^5} + \frac{243}{14}\frac{\left(-x_n + x\right)^8}{h^6}$$

$$\beta_{\frac{1}{2}} = -\frac{197}{2835}\left(-x_n + x\right)h + \frac{20}{3}\frac{\left(-x_n + x\right)^3}{h} - \frac{127}{3}\frac{\left(-x_n + x\right)^4}{h^2} + \frac{558}{5}\frac{\left(-x_n + x\right)^5}{h^3} - \frac{726}{5}\frac{\left(-x_n + x\right)^6}{h^4} + \frac{648}{7}\frac{\left(-x_n + x\right)^7}{h^5} - \frac{162}{7}\frac{\left(-x_n + x\right)^8}{h^6}$$

$$\beta_{\frac{2}{3}} = \frac{97}{2520}\left(-x_n + x\right)h - \frac{15}{4}\frac{\left(-x_n + x\right)^3}{h} + \frac{99}{4}\frac{\left(-x_n + x\right)^4}{h^2} - \frac{2763}{40}\frac{\left(-x_n + x\right)^5}{h^3} + \frac{963}{10}\frac{\left(-x_n + x\right)^6}{h^4} - \frac{459}{7}\frac{\left(-x_n + x\right)^7}{h^5} + \frac{243}{14}\frac{\left(-x_n + x\right)^8}{h^6}$$

$$\beta_{\frac{5}{6}} = -\frac{23}{1890}\left(-x_n + x\right)h + \frac{6}{5}\frac{\left(-x_n + x\right)^3}{h} - \frac{81}{10}\frac{\left(-x_n + x\right)^4}{h^2} + \frac{117}{5}\frac{\left(-x_n + x\right)^5}{h^3} - \frac{171}{5}\frac{\left(-x_n + x\right)^6}{h^4} + \frac{864}{35}\frac{\left(-x_n + x\right)^7}{h^5} - \frac{243}{35}\frac{\left(-x_n + x\right)^8}{h^6}$$

$$\beta_1 = \frac{19}{11340}\left(-x_n + x\right)h - \frac{1}{6}\frac{\left(-x_n + x\right)^3}{h} + \frac{137}{120}\frac{\left(-x_n + x\right)^4}{h^2} - \frac{27}{8}\frac{\left(-x_n + x\right)^5}{h^3} + \frac{51}{10}\frac{\left(-x_n + x\right)^6}{h^4} - \frac{27}{7}\frac{\left(-x_n + x\right)^7}{h^5} + \frac{81}{70}\frac{\left(-x_n + x\right)^8}{h^6}$$

Equation (6) is evaluated at the non-interpolating points $\left\{x_{n+\frac{1}{6}}, x_{n+\frac{1}{2}}, x_{n+\frac{2}{3}}, x_{n+\frac{5}{6}}, x_{n+1}\right\}$ and (7) at all points $\left\{x_n, x_{n+\frac{1}{6}}, x_{n+\frac{1}{3}}, x_{n+\frac{1}{2}}, x_{n+\frac{2}{3}}, x_{n+\frac{5}{6}}, x_{n+1}\right\}$, which produces the following general equations in block form

$$A^{(0)}Y_m^{(i)} = \sum_{i=0}^{1} h^i e_i y_n^i + h^2 b_i f\left(y_n\right) + h^2 d_i f\left(y_m\right) \tag{8}$$

Where

$$Y_m = \left[y_{n+\frac{1}{6}}, y_{n+\frac{1}{3}}, y_{n+\frac{1}{2}}, y_{n+\frac{2}{3}}, y_{n+\frac{5}{6}}, y_{n+1} \right], \quad f(y_m) = \left[f_{n+\frac{1}{6}}, f_{n+\frac{1}{3}}, f_{n+\frac{1}{2}}, f_{n+\frac{2}{3}}, f_{n+\frac{5}{6}}, f_{n+1} \right]$$

$$y_n^i = \left[y_{n-\frac{5}{6}}, y_{n-\frac{2}{3}}, y_{n-\frac{1}{2}}, y_{n-\frac{1}{3}}, y_{n-\frac{1}{6}}, y_n \right], \quad f(y_n) = \left[f_{n-\frac{5}{6}}, f_{n-\frac{2}{3}}, f_{n-\frac{1}{2}}, f_{n-\frac{1}{3}}, f_{n-\frac{1}{6}}, f_n \right]$$

and $A^{(0)} = 6\times 6$ identity matrix ,

when i=0

$$e_0 = \begin{bmatrix} 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0&0&0&0&0&\frac{1}{6} \\ 0&0&0&0&0&\frac{1}{3} \\ 0&0&0&0&0&\frac{1}{2} \\ 0&0&0&0&0&\frac{2}{3} \\ 0&0&0&0&0&\frac{5}{6} \\ 0&0&0&0&0&1 \end{bmatrix}, \quad b_0 = \begin{bmatrix} 0&0&0&0&0&\frac{28549}{4354560} \\ 0&0&0&0&0&\frac{1027}{68040} \\ 0&0&0&0&0&\frac{253}{10752} \\ 0&0&0&0&0&\frac{272}{8505} \\ 0&0&0&0&0&\frac{35225}{870912} \\ 0&0&0&0&0&\frac{41}{840} \end{bmatrix},$$

$$d_0 = \begin{bmatrix} \frac{275}{20736} & \frac{-5717}{483840} & \frac{10621}{1088640} & \frac{-7703}{1451520} & \frac{403}{241920} & \frac{-199}{870912} \\ \frac{97}{1890} & \frac{-2}{89} & \frac{197}{8505} & \frac{-97}{7560} & \frac{23}{5670} & \frac{-19}{34020} \\ \frac{165}{1792} & \frac{-267}{17920} & \frac{5}{128} & \frac{-363}{17920} & \frac{57}{8960} & \frac{-47}{53760} \\ \frac{376}{2835} & \frac{-2}{945} & \frac{656}{8505} & \frac{-2}{81} & \frac{8}{945} & \frac{-2}{1701} \\ \frac{8375}{48384} & \frac{3125}{290304} & \frac{25625}{217728} & \frac{-625}{96768} & \frac{275}{20736} & \frac{-1375}{870912} \\ \frac{3}{14} & \frac{3}{140} & \frac{17}{105} & \frac{3}{280} & \frac{3}{70} & 0 \end{bmatrix}$$

when i=1

$$e_0 = \begin{bmatrix} 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \end{bmatrix}, \quad , \quad b_0 = \begin{bmatrix} 0&0&0&0&0&\frac{19087}{362880} \\ 0&0&0&0&0&\frac{1139}{22680} \\ 0&0&0&0&0&\frac{137}{2688} \\ 0&0&0&0&0&\frac{143}{2835} \\ 0&0&0&0&0&\frac{3715}{72576} \\ 0&0&0&0&0&\frac{41}{840} \end{bmatrix},$$

$$d_0 = \begin{bmatrix} \frac{2713}{15120} & \frac{-15487}{120960} & \frac{293}{2835} & \frac{-6737}{120960} & \frac{263}{15120} & \frac{-863}{362880} \\ \frac{47}{189} & \frac{11}{7560} & \frac{166}{2835} & \frac{-269}{7560} & \frac{11}{945} & \frac{-37}{22680} \\ \frac{27}{112} & \frac{387}{4480} & \frac{17}{105} & \frac{-243}{4480} & \frac{9}{560} & \frac{-29}{13440} \\ \frac{232}{945} & \frac{64}{945} & \frac{752}{2835} & \frac{29}{945} & \frac{8}{945} & \frac{-4}{2835} \\ \frac{725}{3024} & \frac{2125}{24192} & \frac{125}{567} & \frac{3875}{24192} & \frac{235}{3024} & \frac{-275}{72576} \\ \frac{9}{35} & \frac{9}{280} & \frac{34}{105} & \frac{9}{280} & \frac{9}{35} & \frac{41}{840} \end{bmatrix}$$

3 Analysis of Basic Properties of the Method

3.1 Order of the block

Let the linear operator associated with the block (8) be defined as

$$L\{y(x);h\} = A^{(0)}Y_m^{(i)} - \sum_{i=0}^{1} h^i e_i y_n^i - h^2 b_i f(y_n) - h^2 d_i f(y_m) \tag{9}$$

Expanding (9) using Taylor series and comparing the coefficients in h gives

$$L\{y(x);h\} = c_0y(x) + c_1hy'(x) + c_2h^2y''(x) + \ldots + c_ph^py^p(x) + c_{p+1}h^{p+1}y^{p+1}(x) + c_{p+2}h^{p+2}y^{p+2}(x)\ldots \qquad (10)$$

Definition 1: The linear operator and the associated continuous linear multistep method (6) are said to of order p if $c_0 = c_1 = c_2 = \ldots = c_p = 0$ $c_{p+1} = 0.\ and\ c_{p+2} \neq 0$, c_{p+2} is called the error constant and the local truncation error is given by

$$t_{n+k} = c_{p+2}h^{(p+2)}y^{(p+2)}(x_n) + o(h^{p+3}) \qquad (11)$$

For our method

$$\begin{bmatrix}
\sum_{j=0}^{\infty}\frac{\left(\frac{1}{6}h\right)^j}{j!}y_n^{(j)} - y_n - \frac{1}{6}hy'_n - \frac{28549}{4354560}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{275}{20736}\left(\frac{1}{6}\right)^j + \frac{5717}{483840}\left(\frac{1}{3}\right)^j - \frac{10621}{1088640}\left(\frac{1}{2}\right)^j + \frac{7703}{1451520}\left(\frac{2}{3}\right)^j - \frac{403}{241920}\left(\frac{5}{6}\right)^j + \frac{199}{870912}(1)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{1}{3}h\right)^j}{j!}y_n^{(j)} - y_n - \frac{1}{3}hy'_n - \frac{1027}{68040}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{97}{1890}\left(\frac{1}{6}\right)^j + \frac{2}{81}\left(\frac{1}{3}\right)^j - \frac{197}{8505}\left(\frac{1}{2}\right)^j + \frac{97}{7560}\left(\frac{2}{3}\right)^j - \frac{23}{5670}\left(\frac{5}{6}\right)^j + \frac{19}{34020}(1)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{1}{2}h\right)^j}{j!}y_n^{(j)} - y_n - \frac{1}{2}hy'_n - \frac{253}{10752}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{165}{1792}\left(\frac{1}{6}\right)^j + \frac{267}{17920}\left(\frac{1}{3}\right)^j - \frac{5}{128}\left(\frac{1}{2}\right)^j + \frac{363}{17920}\left(\frac{2}{3}\right)^j - \frac{57}{8960}\left(\frac{5}{6}\right)^j + \frac{47}{53760}(1)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{2}{3}h\right)^j}{j!}y_n^{(j)} - y_n - \frac{2}{3}hy'_n - \frac{272}{8505}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{376}{2835}\left(\frac{1}{6}\right)^j + \frac{2}{945}\left(\frac{1}{3}\right)^j - \frac{656}{8505}\left(\frac{1}{2}\right)^j + \frac{2}{81}\left(\frac{2}{3}\right)^j - \frac{8}{945}\left(\frac{5}{6}\right)^j + \frac{2}{1701}(1)^j\right] \\
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{6}h\right)^j}{j!}y_n^{(j)} - y_n - \frac{5}{6}hy'_n - \frac{35225}{870912}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{8375}{48384}\left(\frac{1}{6}\right)^j + \frac{3125}{290304}\left(\frac{1}{3}\right)^j + \frac{25625}{217728}\left(\frac{1}{2}\right)^j - \frac{625}{96768}\left(\frac{2}{3}\right)^j + \frac{275}{20736}\left(\frac{5}{6}\right)^j - \frac{1375}{870912}(1)^j\right] \\
\sum_{j=0}^{\infty}\frac{(1)^j}{j!}y_n^{(j)} - y_n - hy'_n - \frac{41}{840}h^2y''_n - \sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{j+2}\left[\frac{3}{14}\left(\frac{1}{6}\right)^j + \frac{3}{140}\left(\frac{1}{3}\right)^j + \frac{17}{105}\left(\frac{1}{2}\right)^j + \frac{3}{280}\left(\frac{2}{3}\right)^j + \frac{3}{70}\left(\frac{5}{6}\right)^j\right]
\end{bmatrix} = 0$$

Comparing the coefficient of h gives $C_0 = C_1 = C_2 = C_3 = \ldots = C_8 = 0$ and

$$C_9 = \left[\frac{6031}{9142485811200}, \frac{233}{142851340800}, \frac{1}{1371686400}, \frac{31}{8928208800}, , \frac{1625}{365699432448}, \frac{1}{195955200},\right]^T$$

3.2 Zero stability of our method

Definition 2: A block method is said to be zero-stable if as $h \to 0$, the root $z_i, i = 1(1)k$ of the first characteristic polynomial $\rho(z) = 0$ that is $\rho(z) = \det\left[\sum_{j=0}^{k}A^{(i)}z^{k-i}\right] = 0$ Satisfies $|z_i| \leq 1$ and for those roots with $|z_i|$ =1, multiplicity must not exceed two. The block method for k=1, with five off grid collocation point expressed in the form

$$\rho(z)=\left| z\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&1&0&0&\frac{h}{6}\\0&0&1&0&0&\frac{h}{3}\\0&0&1&0&0&\frac{h}{2}\\0&0&0&0&0&\frac{2h}{3}\\0&0&0&0&0&\frac{5h}{6}\\0&0&0&0&0&1\end{bmatrix}\right| = z^4(z-1)^2$$

$$\rho(z)=z^4(z-1)^2=0,$$

Hence, our method is zero-stable.

3.3 Regions of absolute stability (RAS)

The stability polynomial for K=1 with five offsite points gives

$$\bar{h}(w)=h^{12}\left(\left(\frac{1}{426649337856}\right)w^6-\left(\frac{157}{4266493378560}\right)w^5\right)-h^{10}\left(\left(\frac{1943}{63489484800}\right)w^5-\left(\frac{131}{338610585600}\right)w^6\right)$$
$$-h^8\left(\left(\frac{301}{37791360}\right)w^5-\left(\frac{89}{10581580800}\right)w^6\right)-h^6\left(\left(\frac{67}{94058496}\right)w^6+\left(\frac{40613}{47029248}\right)w^5\right)-h^4\left(\left(\frac{6137}{135520}\right)w^5-\left(\frac{41}{311040}\right)w^6\right)$$
$$-h^2\left(\left(\frac{7}{432}\right)w^6+\left(\frac{137}{216}\right)w^5\right)+w^6-2w^5$$

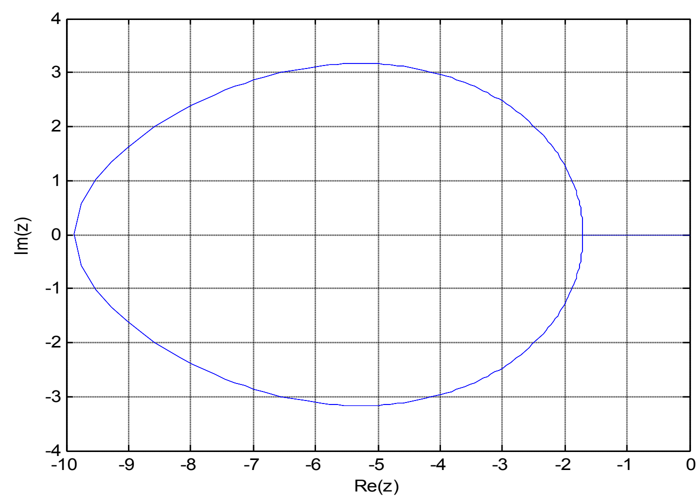

Fig. 1.

3.4 Numerical example

Problem I. We consider a highly stiff problem

$$y''+1001y'+1000y=0,\quad y(0)=1, y'(0)=-1$$

Exact Solution: $y(x)=\exp(-x)\; h=\frac{1}{10}$

Table 1. Comparison of the proposed method with [3]

x-values	Exact solution	Computed solution	Error in our method	Error in [3]
0.100	0.90483741803595957316	0.90483741803595957264	5.20000E(-19)	6.300E(-19)
0.200	0.81873075307798185867	0.81873075307798185765	1.02000E(-18)	4.981E(-17)
0.300	0.74081822068171786607	0.74081822068171786461	1.46000E(-18)	4.902E(-19)
0.400	0.67032004603563930074	0.67032004603563929894	1.8000E(-18)	1.1284E(-16)
0.500	0.60653065971263342360	0.60653065971263342153	2.0700E(-18)	2.7737E(-16)
0.600	0.54881163609402643263	0.54881163609402643035	2.2800E(-18)	4.9717E(-16)
0.700	0.49658530379140951470	0.49658530379140951227	2.43000E(-18)	8.0345E(-16)
0.800	0.44932896411722159143	0.44932896411722158889	2.5400E(-18)	1.16078E(-15)
0.900	0.40656965974059911188	0.40656965974059910927	2.6100E(-18)	1.59645E(-15)
1.00	0.36787944117144232160	0.36787944117144231895	2.65000E(-18)	2.0833E(-15)

Problem II. We consider the second order ODE

$$y'' = 8y' - 17y, \quad y(0) = -4, y'(0) = -1$$

Exact Solution: $y(x) = -4e^{4x}\cos(x) + 15e^{4x}\sin(x), \quad 0 \le x \le 1, \quad h = \frac{1}{100}$

Table 2. Comparison of the proposed method with [13]

x-values	Exact solution	Computed solution	Error in our method	Error in [13] for Bhy NM 2
0.01	-4.00691592223446041400	-4.00691592223446041400	0.000000	7.000E(-13)
0.02	-4.00731721493851945550	-4.00731721493851945540	1.00E(-19)	1.500E(-12)
0.03	-4.00066058359423274020	-4.00066058359423274000	2.00E(-19)	2.390E(-12)
0.04	-3.98636997397524125520	-3.98636997397524125510	1.00E(-19)	3.390E(-12)
0.05	-3.96383486307199070650	-3.96383486307199070640	1.00E(-19)	4.500E(-12)
0.06	-3.93240846866906546580	-3.93240846866906546560	1.00E(-19)	5.730E(-12)
0.07	-3.89140587396774133530	-3.89140587396774133510	2.00E(-19)	7.090E(-12)
0.08	-3.84010206349678951350	-3.84010206349678951340	1.00E(-19)	8.590E(-12)
0.09	-3.77772986639864159710	-3.77772986639864159680	3.00E(-19)	1.023E(-11)
0.10	-3.70347780301604081920	-3.70347780301604081920	0.0000000	1.203E(-11)

Problem III. $f(x, y, y') = y', \quad y(0) = 1, \ y'(0) = \frac{1}{2}, \ 0 \le x \le 1.$

Exact Solution: $y(x) = 1 - e^{x} \ \textit{with} \ h = \frac{1}{100}$

Table 3. Comparison of the proposed method with [14]

x-values	Exact solution	Computed solution	Error in our method	Error in [14]
0.01	-0.10517091807564761917	-0.1051709180756476248	5.630E(-18)	8.326673E(-17)
0.02	-0.22140275816016982146	-0.22140275816016983390	1.244E(-17)	2.775558E(-16)
0.03	-0.34985880757600308334	-0.34985880757600310400	2.066E(-17)	5.551115E(-16)
0.04	-0.49182469764127028742	-0.49182469764127031780	3.038E(-17)	9.436896E(-16)
0.05	-0.64872127070012810487	-0.64872127070012814680	4.193E(-17)	2.109424E(-15)
0.06	-0.82211880039050891923	-0.82211880039050897490	5.567E(-17)	3.219647E(-15)
0.07	-1.01375270747047644990	-1.01375270747047652160	7.170E(-17)	4.440892E(-15)
0.08	-1.22554092849246751400	-1.22554092849246760460	9.060E(-17)	5.995204E(-15)
0.09	-1.45960311115694955120	-1.45960311115694966380	1.126E(-16)	7.771561E(-15)
0.10	-1.71828182845904509720	-1.71828182845904523540	1.382E(-16)	1.065814E(-15)

4 Conclusions

From the above tables, it shows that our proposed methods are indeed accurate, and can handle stiff equations.

Comparing the new method with the existing method [3,14,13,15], the result presented in the tables 1, 2 and 3 respectively shows that the new method performs better than the existing method.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Omar Z, Suleiman M. Parallel r-point implicit block method for solving higher order ordinary differential equations directly. J. ICT. 2004;3:53-66.

[2] Kayode SJ, Adeyeye O. A 3-step hybrid method for direct solution of second order initial value problems. Aust. J. Basic Appl. Sci. 2011;5:2121-2126.

[3] Raymond D, Skwame Y, Joshua S. An implicit two-step one off-grid point third derivative hybrid block method for the direct solution of second order ordinary differential equations. 2018;4(2):8-14.

[4] Bun RA, Vasil's TD. A numerical method for solving differential equations of any order. Comp. Math. Phys. 1992;32(3):317-330.

[5] Fatunla SO. Block methods for second order IVPs. Int. J.Comput. Maths. 1991;41:55-63J.

[6] Vigo-Aguilar, Ramos H. Variable stepsize implementation of multistep methods for $y'' = f(x, y, y^1)$. J. Comput. Appl. Math. 2006;192:114–131.

[7] Adeniran AO, Ogundare BS. An efficient hybrid numerical scheme for solving general second order initial value problems (IVPs). International Journal of Applied Mathematical Research. 2015;4(2): 411-419.

[8] Abdelrahim R, Zurni O. Direct solution of second-order ordinary differential equation using a single-step hybrid block method of order five. Mathematical and Computational Applications. 2016;21:12.

[9] Mohammad A, Zurni O. Implicit one-step block hybrid third-derivative method for the direct solution of initial value problems of second-order ordinary differential equations. Hindawi Journal of Applied Mathematics; 2017. Article ID 8510948.
Available:https://doi.org/10.1155/2017/8510948

[10] Jator SN. A six order linear multi step method for the direct solution of $y'' = f(x, y, y^1)$. Intern. J. Pure and Applied of Maths. 2007;40(1):457-472.

[11] Jator SN, Li J. A self-starting linear multistep method for direct solution general second order initial value problems. Intern. J. Compu. Math. 2009;86(5):817-836.
DOI: 10.1080/00207160701708250

[12] Majid ZA, Sulaiman MB, Omar Z. 3 points implicit block method for solving ordinary differential equations. Bull. Malaysia Mathematical Science Soc. 2006;29(1) 23-31.

[13] Awari YS. Some generalized two-step block hybrid Numerov method for solving general second order ordinary differential equations without predictors. Science World Journal. 2017;12(4):12-18.

[14] Alkasassbeh M, Omar Z. Generalized two-hybrid one-step implicit third derivatives block method for the direct solution of second order ordinary differential equations. International Journal of Pure and Applied Mathematics. 2017;112(3):497-517.

[15] Skwame Y, Raymond D. A class of one-step hybrid third derivative block method for the direct solution of initial value problems of second-order ordinary differential equations. International Journal of Engineering and Applied Sciences. 2018;5(1).

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