

Development of Ant Colony Optimization Software as a Solid Waste Management System

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Authors' contributions

This work was carried out in collaboration between all authors. Author OB designed the study, carried out site visits, computational aspects, wrote the protocol and the first draft of the manuscript. Author ILN served as main supervisor of the study, provided guidance on data collection, analysis and modeling, confirmed the accuracy of the results and documentation. Author JCA served as co-supervisor, assisted in initial design of study, and guided in data collection. All authors read and approved the final manuscript.

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ABSTRACT

In this paper a Decision Support System named *W8st colSoft* is developed by employing dynamic programming and swarm intelligence model encoded in Visual Basic Studio 10.0. Solved solutions from literatures were used to validate the developed decision support system. The results obtained from these validation presented an average error margin of 2.58% when compared with that from literature. Also, in order to present the scalability of the swarm intelligence model employed in the developed decision support system, it was used as a decision tool to analyze the collection of solid waste of the University of Port Harcourt three campuses as a whole, unlike recent publication where it was analyzed Campus-wise. The resultant optimal path from the analysis presented a total distance of 15,682 m saving a total distance of 17.15 m when compared with other route options. Additionally, an Evolutionary Algorithm in Microsoft Excel 2013 was applied to the University of Port

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Harcourt four campus segments and the results were compared with those of the Proposed model. The percentage error margin between Evolutionary Algorithm and the Proposed model prediction ranges from -0.34 to 11.27. The Proposed model was able to achieve optimum value with minimum number of iterations in all cases and this is an advantage.

Keywords: Decision support system; W8st colSoft; swarm intelligence; dynamic programming.

1. INTRODUCTION

Decision support systems are exclusive class of computerized information system that registers valuable information from primary data, documents personal knowledge and models used as a tool to identify, solve problems and suggest appropriate decisions in a structured and logical way based on scientific facts [1]. Decision Support Systems also referred to as 'system analysis platforms' [2] can be considered as computer-based systems developed to assist decision makers in problem solving by employing combination of simulation models (SMs), expert systems(ESs) and geographic information systems (GIS) [3].

Chang and Wang [4] made the first attempt to introduce decision support system as a tool to aid management in planning of solid waste collection, recycling and incineration system. Abeliotis and Others [5] grouped Decision Support Systems (DSS) developed for solid waste management into two main categories:

- i. DSS based on applied mathematics, emphasizes application of statistical, optimisation or simulation modelling; and
- ii. DSS that provides specific problem solving expertise stored as facts, rules, and procedures.

Abeliotis and Others [5] further recognised that there are other DSS that use hybrid approaches in providing insights toward efficient solid waste management system.

From researches [6,7,8,9,10], the development of a decision support system to optimize the collection of solid waste which gulps about 60 - 95% of the total finance budgeted by waste management agencies will aid management in making cost effective decisions and plans.

2. DEVELOPMENT OF DECISION SUPPORT TOOL (W8st colSoft)

The development of the decision support tool (W8st colSoft) involved the application of Floyd-Warshall's Algorithm (a dynamic programming

model) and Ant Colony Optimization Algorithm (a swarm intelligence model).

2.1 Floyd-Warshall's Algorithm (Dynamic Programming)

Dynamic programming is a numerical programming method commonly used in solving complex problems optimally piece-wise. It combines knowledge of immediate penalty of a decision at hand with knowledge of the future penalties resulting from the immediate decision. It is both a mathematical optimization method and a computer programming method [11]. This numerical technique can be applied to any problem that requires decisions to be made in stages with the objective of finding a minimal penalty decision pathway [12].

A problem must possess an optimal substructure; and overlapping sub-problems for it to be solved using dynamic programming algorithm. An optimal substructure entails that the solution to the optimization problem can be obtained by the combination of the optimal solution of its sub-problems. Overlapping sub-problems means that any recursive algorithm solving a sub-problem should be able to solve the other sub-problems instead of generating another new algorithm for the other sub-problems. Dynamic programming has two basic approaches for its implementation; the "top-down approach" and the "bottom-up approach" [13]. The basic steps in developing a dynamic programming algorithm are:

- a) Characterize the structure of an optimal solution;
- b) Recursively define the value of the Optimal solution;
- c) Compute the value of the optimal solution, usually in a bottom-up approach; and
- d) Construct an optimal solution from computed information.

Floyd-Warshall's algorithm is a dynamic programming method and it is an algorithm that usually operates on graphs [13]. Its penalty function is the distance between pairs of nodes in

a given weighted graph $G = f(V, E)$. The optimal solution of the sub-problems is to obtain the shortest distance between pair of nodes in the given weighted graph. The associated distance between the nodes in the graph represents the weighted function for the edges.

In the model development of this research, the node represents solid waste collection points and dumpsites while the weighted function for the edges which connects the nodes represents the relative distance between the solid waste collection points and dumpsites.

2.1.1 Developing a dynamic program for shortest-route problem using Floyd-Warshall’s algorithm

The various steps involved in developing a dynamic program for shortest –route problem using Floyd-Warshall’s Algorithm includes:

Step 1- Structure Characterization:

Assume that the nodes of a given weighted graph, $G = f(V, E)$ are

$$V = \{1, 2, 3, \dots, n\}.$$

Let a subset of some nodes say $K = \{1, 2, 3, \dots, k\}$.

For any pair of nodes $i, j \in V$, consider all paths from i to j with intermediate nodes belonging to the subset $\{1, 2, 3, \dots, k\}$ and let the minimum weight path among these nodes be p .

Floyd-Warshall’s algorithm works in a way that it exploits the relationship that exist between the minimum weighted path p and the shortest paths from i to j with all the intermediate nodes in the subset $\{1, 2, 3, \dots, k\}$. The relationship constraint is on the condition of k being an intermediate node of the path p or not:

- i. If k is not an intermediate node of path p , then all intermediate nodes of path p are elements of the set $\{1, 2, 3, \dots, k-1\}$. Thus, a shortest path from nodes i to j with all intermediate nodes in the set $\{1, 2, 3, \dots, k-1\}$ is also a shortest path from i to j with intermediate nodes in the set $\{1, 2, 3, \dots, k\}$.
- ii. If k is an intermediate node of path p , then the path p is broken down or say decomposed into two paths p_1 and p_2 ,

where $p_1 = i \rightsquigarrow k$ and $p_2 = k \rightsquigarrow j$. That is path p will then be:

$$p = p_1 + p_2 = i \rightsquigarrow k \rightsquigarrow j$$

It follows that p_1 is the shortest path from i to k having all intermediate nodes as elements of the set $\{1, 2, 3, \dots, k-1\}$, also p_2 is the shortest path from k to j with all intermediate nodes in the set $\{1, 2, 3, \dots, k-1\}$.

Step 2 – Definition of the value of the optimal solution recursively:

From nodes i to j for which all intermediate nodes are elements of the set $\{1, 2, 3, \dots, k\}$, let $d_{ij}^{(k)}$ be the weight factor. When $k = 0$, the path from nodes i to j has no intermediate nodes numbered higher than 0. It implies that such path has at most one edge and hence, $d_{ij}^{(0)} = w_{ij}$, where: w_{ij} is the value of the weight factor with respect to the edge between nodes i & j .

Defining $d_{ij}^{(k)}$ recursively with respect to the above statements will be:

$$d_{ij}^k = \begin{cases} w_{ij} & \text{if } k = 0. \\ \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}) & \text{if } k \geq 1. \end{cases} \quad (1)$$

Note: For any path, all intermediate nodes are elements in the set $\{1, 2, 3, \dots, n\}$, hence the matrix $D^{(n)} = (d_{ij}^{(n)})$ gives the final solution: $d_{ij}^{(k)} = \delta(i, j)$ for all $i, j \in V$.

Step 3 – Compute the value of the optimal solution (bottom-up approach):

Using Equation (1), compute the values of $d_{ij}^{(k)}$ in order of increasing values of k using the bottom up approach (Table 1). The input used by this approach is a $n \times n$ matrix W is presented as Equation (2):

$$w_{ij} = \begin{cases} 0 & \text{if } i = j, \\ \text{the weight of directed edge } (i, j) & \text{if } i \neq j \text{ and } (i, j) \in E, \\ \infty & \text{if } i \neq j \text{ and } (i, j) \notin E. \end{cases} \quad (2)$$

Note: From Equation (2) it follows that if there is no direct path connecting nodes i to j the weight of the edge between the nodes = ∞ (or say NIL).

Table 1. Final computation of the value of the optimal solution

```

Input initial  $n \times n$  matrix  $W$  (see Equation (2))
 $D^{(0)} = W$ 
for  $k=1$  to  $n$ 
  let  $D^{(k)} = (d_{ij}^{(k)})$  be a new  $n \times n$  matrix
  for  $i=1$  to  $n$ 
    for  $j=1$  to  $n$ 
       $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$ 
  Return  $D^{(n)}$  (optimal solution i.e. Matrix of pairs of shortest path weight).

```

Step 4 – Construction of Optimal Solution:

The optimal solution is constructed based on the computed information from *step 3*. The pseudo code for the construction of the optimal solution is presented in Table 2. The code assumes an input graph of N nodes.

2.2 Swarm Intelligence (SI)

Swarm Intelligence is a group of computational intelligence that represents algorithms developed or modeled from the study of swarming behaviors in nature such as the social behavior of organisms in swarm [14]. It uses group of simple individuals to solve complex problems through simple and binary interactions. Swarm Intelligence has a goal to design intelligent multi-agent systems by taking inspiration from the collective behavior of social insects, for example, the social and foraging behavior of ants in colonies; the bee colony; the graceful, but unpredictable, choreography of bird flocks; school of fish; and aggregating slime molds. Swarm Intelligence algorithms are most useful

for problems that are dynamic in nature and are amendable to an agent based decomposition.

2.2.1 Ant Colony Optimization (ACO)

Biologists have proven that the self-organizing behavior of social insects can be presented through simple models using only *stigmergic* interaction [15]. Ant colony optimization involves a colony of artificial ants which are abstractions from the behavioral traits of real ants cooperating in finding better solutions to different discrete optimization problems. The similarities between real ants and artificial ants which is exploited in ACO [16] include:

- i. The use of a colony of cooperate individual into finding a better global solution to the problem/task under consideration;
- ii. The use of artificial pheromone trail for *stigmergic* interaction;
- iii. The use of a local move to find the shortest path between the origin (nest) and the destination (food site); and
- iv. The use of a probabilistic decision policy to move through adjacent nodes.

Table 2. Pseudo code for optimal solution construction

```

Start
  for  $i = 1$  to  $n$ 
    for  $j = 1$  to  $n$ 
      if there is an edge from  $i$  to  $j$ 
         $d_{ij}^{(0)} = w_{ij}$ 
      else
         $d_{ij}^{(0)} = \infty$ 
    for  $k = 1$  to  $n$ 
      for  $i = 1$  to  $n$ 
        for  $j = 1$  to  $n$ 
           $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$ 
  End

```

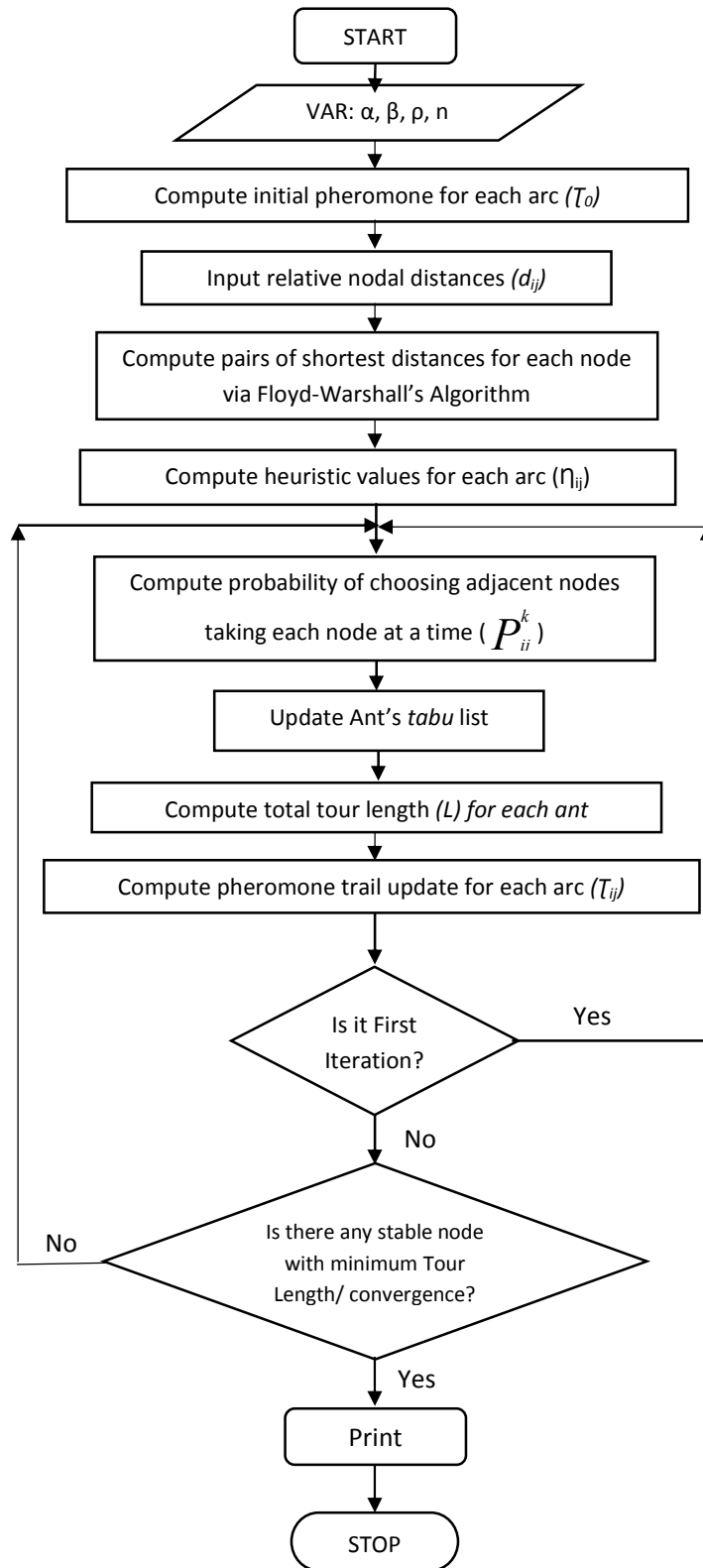


Fig. 1. Flow chart of ACO algorithm [17]

The Ant Colony Optimization algorithm has two basic components [17]:

- i. Pheromone trail on arc $(i, j), T_{ij}$; and
- ii. Heuristic value of arc $(i, j), \eta_{ij}$. This has to do with the desirability of the arc.

In ant colony optimization algorithm, every step in which all the ants involved in the problem solution achieves a feasible solution is referred to as an *iteration*. The flow chart for the Ant Colony Optimization algorithm used for the development of the decision support system (*W8st col/Soft*) is presented in Fig. 1. With reference to the developed decision support system, each node in the program represents waste collection point/dumpsite while the connecting arc represents the route connecting two collection points/dumpsites.

The program commences with the specification of the various variables (α, β, ρ, n) . α & β are positive parameters assigned values 1 & 2 respectively to control the relative weight of *pheromone* information T_{ij} . ρ is the *pheromone* evaporation parameter which usually ranges between 0 & 1 but for the case of the developed decision support system was assigned the value

of 0.5. n represents the number of waste collection points (nodes).

3. DESCRIPTION OF THE DEVELOPED DECISION SUPPORT SYSTEM

The input data used by the developed decision support system is mainly the distances/positions of solid waste collection points/dumpsites. The decision support system employs mainly two models (Dynamic programming model and Swarm intelligence model) encoded in Visual Basic Studio 2010. The developed decision support system has the property that supports the importation of files (usually maps of road network) with file extension of *jpg, jpeg, bmp,* and *png* which could be used as the background to aid the easy location of waste collection points/dumpsites. These collection points are connected with straight lines which when right clicked on opens a property table where the relative distance of the connecting route between the waste collection points are entered (see Fig. 2). This property includes the route name (or street name), the relative distance (in metres) between collection points, the traffic flow direction through the route (that is, one way or two way traffic flow direction(s)).

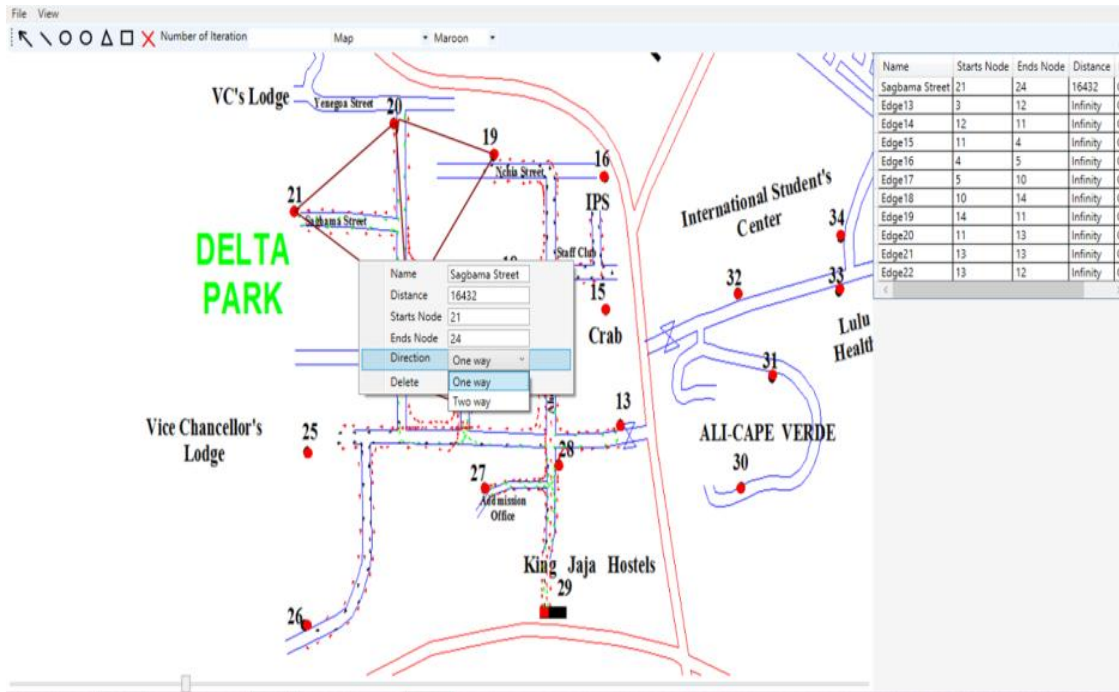


Fig. 2. Display screen of decision support system when performing an analysis

The tool bar consists of icons (from left to right) whose properties include, selection of elements, placement of collection points, deleting of elements, number of iterations and colour assignment for collection points, connecting arc and background input properties. Also on the far end of the display screen is a table being formed as the collection points are placed and assigned properties (see Fig. 2).

3.1 Solving a Hypothetical Problem Using *W8st colSoft*

A sketch of the road network of an estate (Fig. 3) with solid waste collection points labeled 1-5 is presented and the management of the estate wants to make the best decision on how to effectively collect the solid waste generated within the proposed estate.

The following step will take one through the basics in applying *W8st colSoft* in solving a problem.

Step 1: Call up the Decision Support System from the installed program on your computer desk top by double clicking;

Step 2: Import map of proposed problem area and insert the collection points (see Fig. 4);

Step 3: Insert the connecting arc to the collection points 1,2,...,5 and left click once and right click also on the connecting arc to open a property box where the details of the arc will be inserted in terms of distance, collection points at its end and beginning, the nature of the route if it is a one way traffic route or a two way(see Fig. 5).Note for this analysis the direction of traffic flow is assumed to be two way; and

Step 4: Click on the View Tab on the Menu bar to view the results of the analysis with respect to the computed initial pheromone, input relative nodal distances, computed pairs of shortest distances, computed heuristic values for each arc, and the Computed tour path (see Fig. 6).

Given: Distance between collection point

Point 1 - Point 2 = 65m

Point 2 – Point 3 = 45m

Point 3 – Pont 4 = 50m

Point 1 – Point 5 = 80m

Point 4 – Point 5= 65m

$\alpha = 1$; $\beta = 2$; and $\rho = 0.5$

where α and $\beta =$ weighting constant

$\rho =$ pheromone evaporation constant

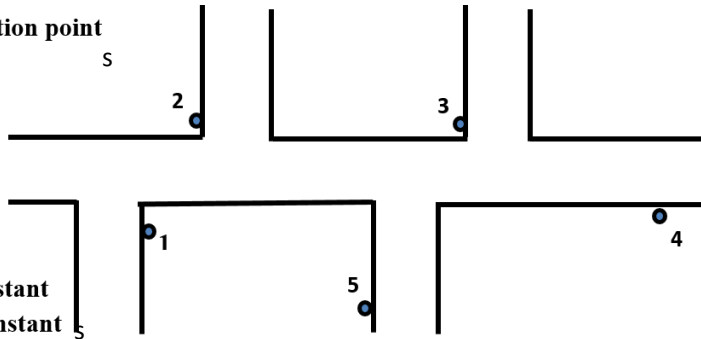


Fig. 3. Road network of hypothetical estate with waste collection points

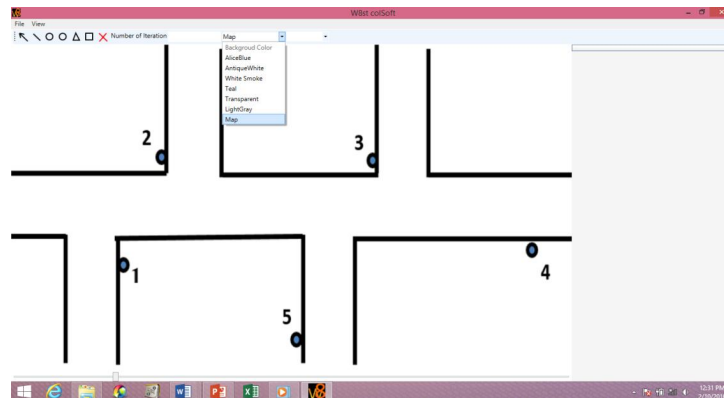


Fig. 4. Screen shot of *w8st colSoft* with map being imported

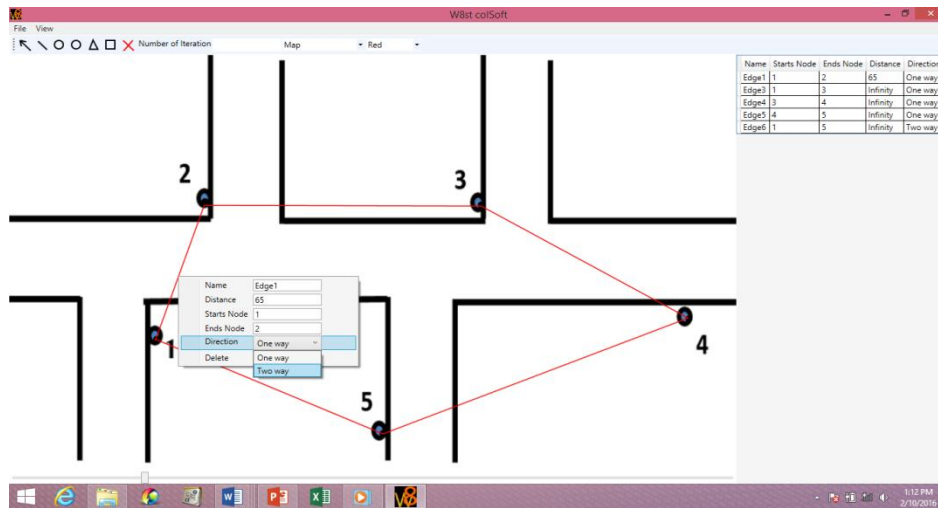


Fig. 5. Screen shot with collection points being connected and assigned properties with respect to the connecting road network

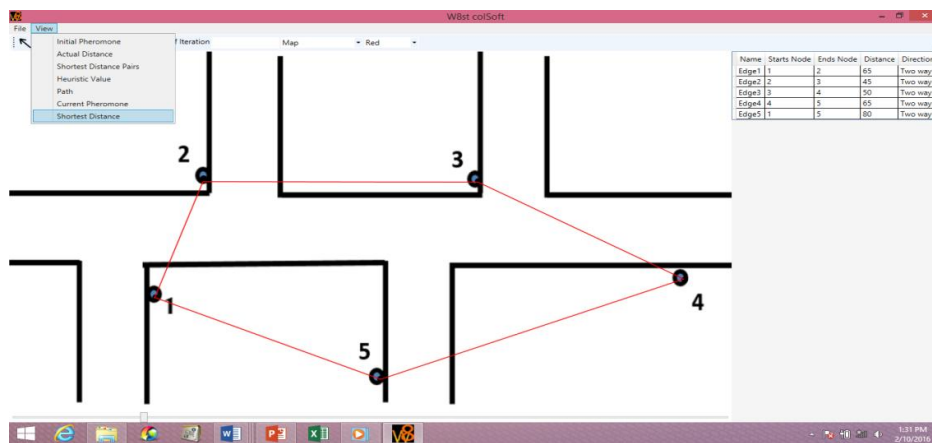


Fig. 6. Screen shot with a drop down menu on resultant outputs from analysis

The resultant outputs from the developed decision support system (*W8st colSoft*) for the hypothetical problem is presented along with the manual calculation for the problem in *Appendixes A1 & A2*.

4. RESULTS AND DISCUSSION

4.1 Results

In order to validate the accuracy of the developed model (*w8st colSoft*), it was necessary to employ illustrative example problems from textbook, case study from literature (University of Port Harcourt example), and a typical example taken from Microsoft Excel 2013. The first example problem exemplified the

application of dynamic programming in finding the shortest distance between two or more nodes [18]; whereas the case study problem was adopted to compare simulated result and manually computed result based on the same principle. However, an Evolutionary Algorithm in Microsoft Excel 2013 was applied to the University of Port Harcourt four campus segments and the results were compared with those of the Proposed model (*w8st colSoft*). The simulated results from both models are as presented later in the discussion section.

The first illustrative example problem taken from a textbook [18] was used to validate the accuracy of the developed decision support system (*W8st colSoft*) (see Fig. 7). The resultant output with regards to the application of Floyd-Warshall's

algorithm (dynamic programming) is presented in Table 3. Also in validating the DST (Decision Support Tool) as regards decision to aid the optimization of solid waste collection, a second solved example problem (see Fig. 8) from a research publication [19] was used and the results are as presented in Table 4. Furthermore, the developed DST was applied to aid decision

to optimize the solid waste route for the University of Port Harcourt as illustrative example 3 [17] and a plot of the resultant output is presented in Fig. 9. The simulated results using evolutionary algorithm and the proposed model for University of Port Harcourt four campus segments are as presented in Table 4 and Figs. 11a-d.

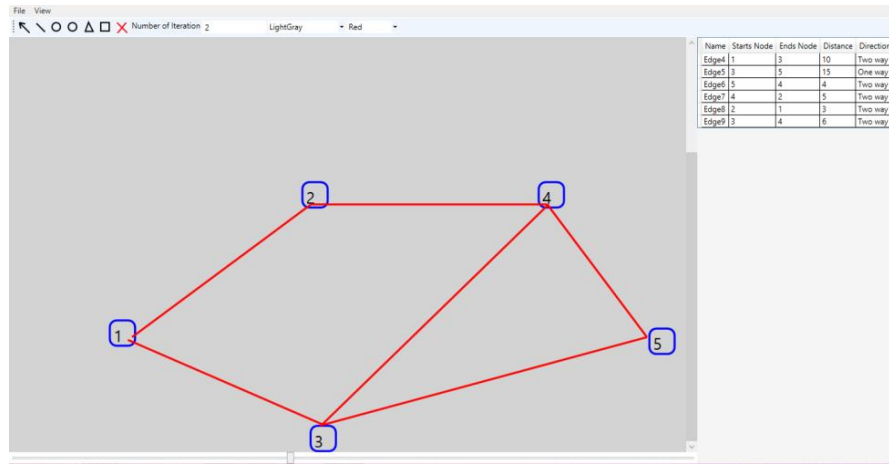


Fig. 7. Screen shot of developed decision system showing the analysis for the validation using solved example from Textbook with regards to Dynamic Programming model

Table 3. Output from *W8st colSoft* using data from textbook exercise [17]

Location	Node1	Node2	Node3	Node4	Node5
Node1	0	3	10	8	12
Node2	3	0	11	5	9
Node3	10	11	0	6	10
Node4	8	5	6	0	4
Node5	12	9	10	4	0

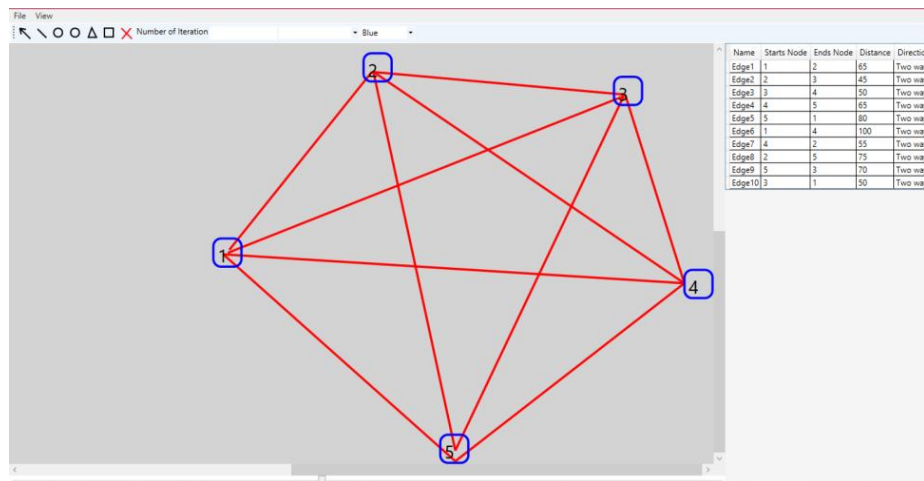


Fig. 8. Screen shot of *W8st colSoft* showing the analysis for the validation using solved example from Publication [19] with regards to Ant Colony Optimization Algorithm

Table 4. W8st colSoft output using data from research publication [19] with regards to Ant colony optimization algorithm as input data

Output	Values	No. of iterations
Research publication	215 m	5
W8st colSoft	215 m	2
Error	0	-

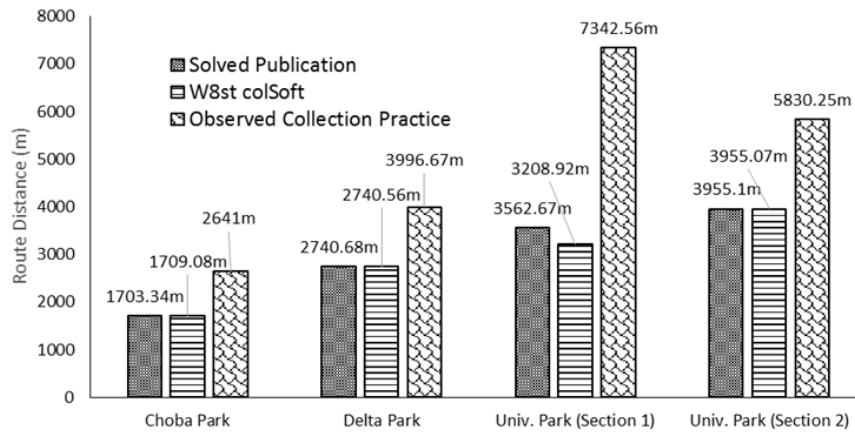


Fig. 9. Plot of the optimal route distance on University of Port Harcourt 3 campuses using W8st colSoft and results of analysis from published literature

In an attempt to present the advantage of the developed decision support system (W8st colSoft) over the analysis made in earlier publication [17] the solid waste collection at the

University of Port Harcourt three campuses clustered together is analyzed as a whole (instead of campus by campus) (see Fig. 10).

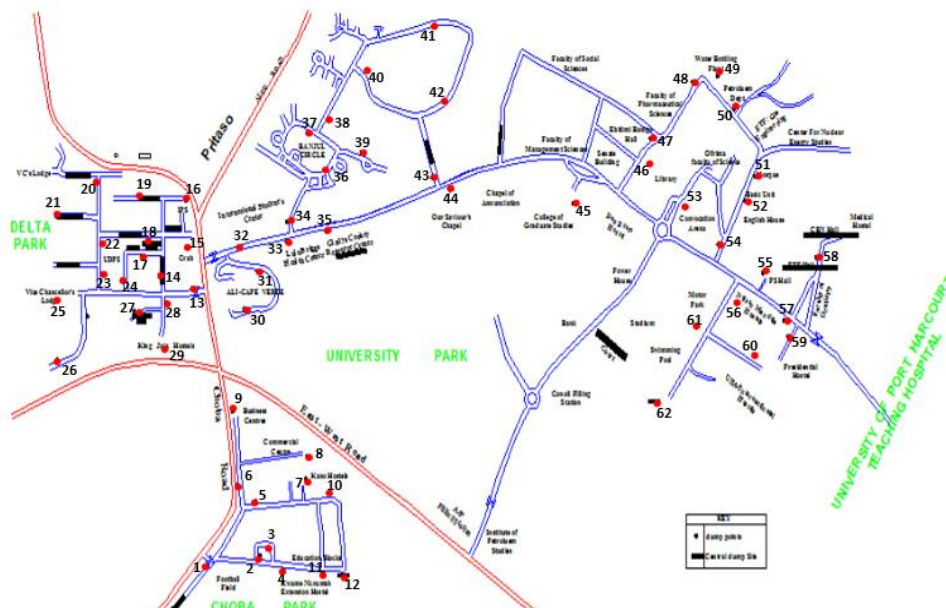


Fig. 10. Road network of the University of Port Harcourt showing solid waste collection points
 Source: Bovwe and Nwaogazie [17]

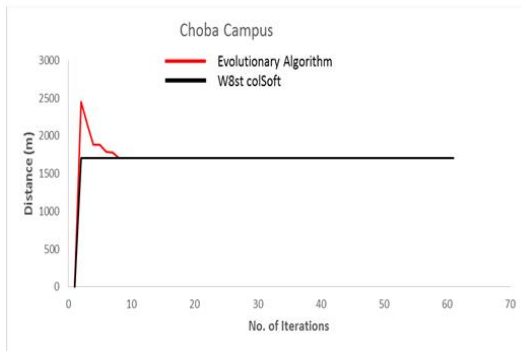
The resultant final optimized route for the collection of the generated solid waste within the University is as shown below (see details in Appendix A):

53=>62=>60=>61=>56=>58=>59=>57=>55=>54=>52=>51=>50=>49=>48=>47=>46=>45=>44=>43=>42=>40=>38=>37=>36=>39=>41=>32=>33=>34=>35=>30=>13=>28=>14=>17=>24=>23=>22=>18=>16=>19=>20=>21=>25=>26=>27=>29=>15=>1=>2=>3=>4=>11=>12=>10=>7 =>5=>6=>9=>8

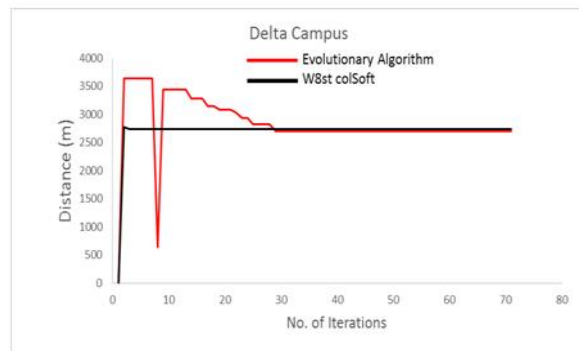
Table 5. Simulated outputs of observed, evolutionary algorithm, proposed model and literature

(1) Campuses	(2) Observed collection practice	(3) Literature [±]	(4) W8st colSoft	(5) Evolutionary algorithm [*]	% Error $\left(\frac{Col\ 5 - Col\ 4}{Col\ 5} \times 100\right)$
Choba Park	2641	1703.34	1709.08	1703.34	-0.337
Delta Park	3996.67	2740.68	2740.56	2702.64	-1.403
Univ. Park (Section 1)	7342.56	3562.67	3208.92	3019.03	-6.290
Univ. Park (Section 2)	5830.25	3955.1	3955.07	4457.18	11.265

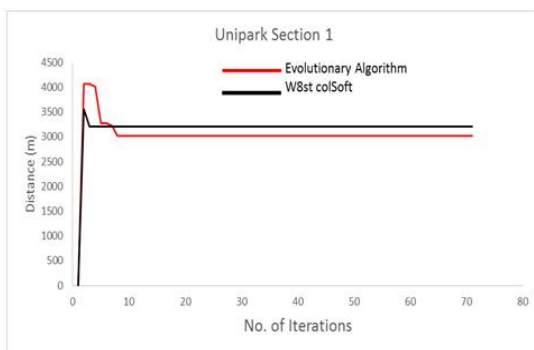
^{*} Microsoft Excel 2013; [±] Journal Article [17]



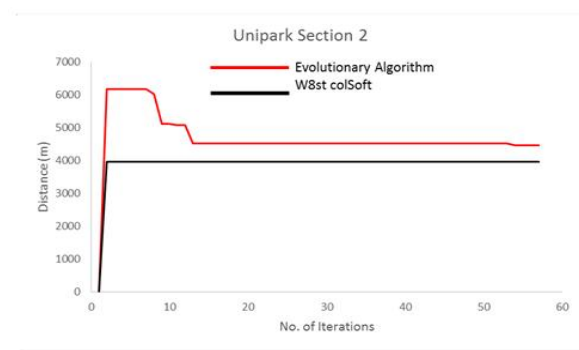
a). EA versus PM for Choba campus



b). EA versus PM for Delta campus



c). EA^{*} versus PM[±] for Unipark campus section 1



d). EA^{*} versus PM[±] for Unipark campus section 2

Fig. 11. Plots of No. of Iterations against output for evolutionary algorithm and proposed model

^{*}EA = Evolutionary Algorithm; [±]PM = Proposed Model (w8st colSoft)

4.2 Discussion

The application of *w8st colSoft* in solving the hypothetical problem when compared with the stress involved in doing the manual calculation (see *Appendixes A1 & A2*) presents a better approach in solving the problem with ease and a higher degree of precision especially when dealing with a more complex network of waste collection problem.

The result from the validation of the developed decision support system (*W8st colSoft*) using example from literature (see Table 4) yielded the same result (215m) at its 2nd iteration unlike the analysis from literature which converged on its 5th iteration. Also as presented in Fig. 10, the decision support system (*W8st colSoft*) was validated using another publication from literature on the optimization of solid waste collection from the University of Port Harcourt. The simulated output from the developed decision system when compared with those from the literature yielded error margins of 0.34, 0.04, 9.93, and 0.00076% for Choba Park, Delta Park and the University Park (sections 1 & 2) campuses, respectively.

The Simulated output from the developed decision support system (*W8st colSoft*) when used as a tool to aid the choice for the optimal routing of solid waste collection within the University of Port Harcourt combining the three Campuses together instead of taking them individually presented an optimal path distance of 15,682m (Node 53) saving a total distance of 17.15m when compared with other route options (see Appendix B).

To add value to the validation of the proposed model it was necessary to compare results from evolutionary algorithm applied to the University of Port Harcourt four campus segments. Both models were allowed to iterate up to 80No. iterations and from Figs. 11a-d the evolutionary simulated results showed early fluctuations between first and thirty iterations before achieving stable and optimum values. However, the proposed model was more stable and achieved optimum solution in less than five iterations. Apparently, the proposed model has an advantage in stability and small number of iterations for optimum solution. At the optimum solution for each of the four campus segments the percentage error between evolutionary algorithm and proposed model is between -0.34 and 11.27. Table 5 equally confirms the close agreement between the results of the proposed

model and those from literature [17]. The reference point in Table 5 is the observed data (collection practice) that is useful in comparing all the simulated options, their effectiveness and the principles in which the models are constructed.

5. CONCLUSION

Based on the study carried out the following conclusions can be drawn:

- i. The proposed model (*W8st colSoft*) makes suggestions of various routes that could be taken with the path gained as a result of each choice.
- ii. The developed model is a very handy tool in the efficient management of solid waste using dynamic and swarm intelligence model;
- iii. Comparison of simulated results using the field conditions of the University of Port Harcourt four campus segments using evolutionary algorithm, proposed model, observed field collection practice (reference values) and literature confirmed the proposed model as very stable with small number of iterations to achieve optimum values;
- iv. The evolutionary algorithm suffers early instability between the first and thirty iterations before achieving steady condition with optimum;
- v. The percent error in the simulated results with evolutionary algorithm and proposed model are between -0.34 and 11.27; and
- vi. The proposed model is a veritable tool for effective solid waste collection and it has shown comparative advantage over evolutionary algorithm in stability and speed (small number of iterations) to achieve optimum values.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX A

A1: Manual calculation of hypothetical problem

The table of relative distance is formed using the given data on the distances between nodes (see Table A1).

Table A1. Initial relative distances between nodes/collection points

Node	1	2	3	4	5
1	0	65	Infinity	Infinity	80
2	65	0	45	Infinity	Infinity
3	Infinity	45	0	50	Infinity
4	Infinity	Infinity	50	0	65
5	80	Infinity	Infinity	65	0

Note: "Nil" is inputted into nodes where there is no direct connectivity. That is, it is assumed that there is a kind of obstruction between the nodes and as such there is no direct connection between them. Also, it is assumed that an ant is placed on each of the nodes. Therefore the initial pheromone $\tau_0 = \frac{1}{n}$, for each node $n = 5$; $\tau_0 = \frac{1}{5} = 0.2$. Therefore forming a initial pheromone matrix (see Table A2):

Table A2. Initial pheromone τ_0

Node	1	2	3	4	5
1	0.2	0.2	0.2	0.2	0.2
2	0.2	0.2	0.2	0.2	0.2
3	0.2	0.2	0.2	0.2	0.2
4	0.2	0.2	0.2	0.2	0.2
5	0.2	0.2	0.2	0.2	0.2

Applying Floyd Warshall's algorithm to compute the pairs of shortest distances (d_{ij}) (see Table A3).

Table A3. Pairs of shortest distances between nodes

Node	1	2	3	4	5
1	0	65	110	145	80
2	65	0	45	95	145
3	110	45	0	50	115
4	145	95	50	0	65
5	80	145	115	65	0

The Heuristic information η_{ij} computed as presented in Table A4 is given by $\eta_{ij} = \frac{1}{d_{ij}}$

Table A4. Computed heuristic information

Node	1	2	3	4	5
1	0	0.015	0.009	0.007	0.013
2	0.015	0	0.022	0.011	0.007
3	0.009	0.022	0	0.020	0.009
4	0.007	0.011	0.020	0	0.015
5	0.013	0.007	0.009	0.015	0

The probability of an ant making its choice on adjacent node to go is given by:

$$P_{ij}^k = \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{i=N_i^k} \tau_{ij}^\alpha \eta_{ij}^\beta}$$

Taking Ant 1: Using the values from Tables A2 & A4, the probability for ant starting at node 1 is:

$$P_{12}^1 = \frac{(0.2)^1(0.015)^2}{[(0.2)^1(0.015)^2] + [(0.2)^1(0.009)^2] + [(0.2)^1(0.007)^2] + [(0.2)^1(0.013)^2]} = 0.429$$

$$P_{13}^1 = \frac{(0.2)^1(0.009)^2}{[(0.2)^1(0.015)^2] + [(0.2)^1(0.009)^2] + [(0.2)^1(0.007)^2] + [(0.2)^1(0.013)^2]} = 0.155$$

$$P_{14}^1 = \frac{(0.2)^1(0.007)^2}{[(0.2)^1(0.015)^2] + [(0.2)^1(0.009)^2] + [(0.2)^1(0.007)^2] + [(0.2)^1(0.013)^2]} = 0.094$$

$$P_{15}^1 = \frac{(0.2)^1(0.013)^2}{[(0.2)^1(0.015)^2] + [(0.2)^1(0.009)^2] + [(0.2)^1(0.007)^2] + [(0.2)^1(0.013)^2]} = 0.323$$

From the above solution the node with the highest probability (*node 2*) is obviously chosen by ant 1 as the adjacent node. From node 2, the probability of ant 1 to choose the next node is given by:

$$P_{23}^1 = \frac{(0.2)^1(0.022)^2}{[(0.2)^1(0.022)^2] + [(0.2)^1(0.011)^2] + [(0.2)^1(0.007)^2]} = 0.740$$

$$P_{24}^1 = \frac{(0.2)^1(0.011)^2}{[(0.2)^1(0.022)^2] + [(0.2)^1(0.011)^2] + [(0.2)^1(0.007)^2]} = 0.185$$

$$P_{25}^1 = \frac{(0.2)^1(0.001)^2}{[(0.2)^1(0.022)^2] + [(0.2)^1(0.011)^2] + [(0.2)^1(0.007)^2]} = 0.075$$

From the above solution the node with the highest probability (*node 3*) is obviously chosen by ant 1 as the adjacent node. From node 3, the probability of ant 1 to choose the next node is given by:

$$P_{34}^1 = \frac{(0.2)^1(0.020)^2}{[(0.2)^1(0.020)^2] + [(0.2)^1(0.009)^2]} = 0.832$$

$$P_{35}^1 = \frac{(0.2)^1(0.009)^2}{[(0.2)^1(0.020)^2] + [(0.2)^1(0.009)^2]} = 0.168$$

From the above result, node 4 has the highest probability, thus Ant 1 moves from node 2 to node 3 and to node 4, then finally node 5 (being the only node left)

Tour path of Ant 1 = **1- 2-3 – 4- 5**

The above analysis is repeated for Ant 2, Ant 3, Ant 4 and Ant 5, respectively. Table A5 presents the resultant of the solution.

Table A5. Summary of computation for hypothetical problem

Ant	Path taken	L (m)	Δτ
1	1-2-3-4-5	225	2.578
2	2-3-4-5-1	240	2.417
3	3-2-1-5-4	255	2.275
4	4-3-2-1-5	240	2.417
5	5-4-3-2-1	225	2.578

A2: output of hypothetical problem from developed decision support system (W8st colSoft) simulation

```

<-----Final Distance Matrix----->
Point          Nodes
Node1  0.00    65.00    110.00   145.00    80.00
Node2  65.00    0.00     45.00    95.00   145.00
Node3  110.00   45.00     0.00    50.00   115.00
Node4  145.00   95.00    50.00     0.00    65.00
Node5   80.00  145.00   115.00    65.00     0.00

Iteration 1
<-----Pheromone----->
Point          Nodes
Node1 0.00  0.20  0.20  0.20  0.20
Node2 0.20  0.00  0.20  0.20  0.20
Node3 0.20  0.20  0.00  0.20  0.20
Node4 0.20  0.20  0.20  0.00  0.20
Node5 0.20  0.20  0.20  0.20  0.00

<-----Path Followed----->
Point          Nodes      Length  Path Gain
Node1 1      2      3      4      5      225.00  2.58
Node2 2      3      4      5      1      240.00  2.42
Node3 3      2      1      5      4      255.00  2.27
Node4 4      3      2      1      5      240.00  2.42
Node5 5      4      3      2      1      225.00  2.58

<-----Minimum Path----->
Point          Nodes      Length  Path Gain
Node1 1      2      3      4      5      225.00  2.58

Iteration 2
<-----Pheromone----->
Point          Nodes
Node1 0.00  9.95  0.10  0.10  7.21
Node2 9.95  0.00  12.36  0.10  0.10
Node3 0.10  12.36  0.00  10.09  0.10
Node4 0.10  0.10  10.09  0.00  9.95
Node5 7.21  0.10  0.10  9.95  0.00

<-----Path Followed----->
Point          Nodes      Length  Path Gain
Node1 1      2      3      4      5      225.00  2.58
Node2 2      3      4      5      1      240.00  2.42
Node3 3      2      1      5      4      255.00  2.27
Node4 4      3      2      1      5      240.00  2.42
Node5 5      4      3      2      1      225.00  2.58

<-----Minimum Path----->
Point          Nodes      Length  Path Gain
Node1 1      2      3      4      5      225.00  2.58

```


APPENDIX B

Output from developed decision support system for analysis of University of Port Harcourt solid waste collection

<-----PATH FOLLOWED----->

NODE	NODES																			
	LENGTH	PATH GAIN																		
Node1	2	3	4	11	12	10	7	5	6	9	8	39	36	34	33	35	32	31	30	
13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	38	37	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	61	60	62		
53	41	17646.33	15.24																	
Node2	3	4	11	12	10	7	5	6	9	8	1	15	29	27	28	14	17	24	23	
22	18	16	19	20	21	25	26	13	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17022.12	15.80																	
Node3	3	2	1	5	6	9	8	7	10	12	11	4	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17339.31	15.51																	
Node4	4	11	12	10	7	5	6	9	8	1	2	3	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17313.56	15.54																	
Node5	5	6	9	8	1	2	3	4	11	12	10	7	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17403.77	15.46																	
Node6	6	5	7	10	12	11	4	3	2	1	15	29	27	28	14	17	24	23	22	18
16	19	20	21	25	26	13	30	32	33	34	35	36	39	38	37	40	42	43	44	
45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	31	
9	8	17972.34	14.97																	
Node7	7	10	12	11	4	3	2	1	5	6	9	8	39	36	34	33	35	32	31	30
13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	38	37	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	
53	41	17652.12	15.24																	
Node8	8	9	6	5	7	10	12	11	4	3	2	1	15	29	27	28	14	17	24	23
22	18	16	19	20	21	25	26	13	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	16621.35	16.18																	
Node9	9	6	5	7	10	12	11	4	3	2	1	15	29	27	28	14	17	24	23	22
18	16	19	20	21	25	26	13	30	32	33	34	35	36	39	38	37	40	42	43	
44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	
31	8	17979.30	14.96																	
Node10	10	7	5	6	9	8	1	2	3	4	11	12	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17406.16	15.45																	
Node11	11	4	3	2	1	5	6	9	8	7	10	12	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17527.00	15.35																	
Node12	12	11	4	3	2	1	5	6	9	8	7	10	13	28	14	17	24	23	22	18
16	19	20	21	25	26	27	29	15	30	32	33	34	35	36	39	38	37	40	42	
43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	31	17445.05	15.42																	
Node13	13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	30	32	33
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	
57	59	58	56	61	60	62	53	41	31	1	2	3	4	11	12	10	7	5	6	
9	8	16840.09	15.97																	
Node14	14	28	13	30	32	33	34	35	36	39	38	37	40	42	43	44	45	46	47	48
49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	31	15	29	27	26	
25	21	20	19	16	18	22	23	24	17	1	2	3	4	11	12	10	7	5	6	
9	8	17091.36	15.74																	
Node15	15	29	27	28	14	17	24	23	22	18	16	19	20	21	25	26	13	30	32	33
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	
57	59	58	56	61	60	62	53	41	31	1	2	3	4	11	12	10	7	5	6	
9	8	16521.43	16.28																	
Node16	16	19	20	21	25	26	27	29	15	17	14	28	13	30	32	33	34	35	36	39
38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	
61	60	62	53	41	31	24	23	22	18	1	2	3	4	11	12	10	7	5	6	
9	8	17075.31	15.75																	
Node17	17	14	28	13	30	32	33	34	35	36	39	38	37	40	42	43	44	45	46	47
48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	31	15	29	27	
26	25	21	20	19	16	18	22	23	24	1	2	3	4	11	12	10	7	5	6	
9	8	17071.68	15.76																	
Node18	18	22	23	24	17	14	28	13	30	32	33	34	35	36	39	38	37	40	42	43
44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	
31	15	29	27	26	25	21	20	19	16	1	2	3	4	11	12	10	7	5	6	
9	8	17203.42	15.64																	
Node19	19	16	18	22	23	24	17	14	28	13	30	32	33	34	35	36	39	38	37	40
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	
53	41	31	15	29	27	26	25	21	20	1	2	3	4	11	12	10	7	5	6	
9	8	17423.58	15.44																	

Node20	20	19	16	18	22	23	24	17	14	28	13	30	32	33	34	35	36	39	38	37	
40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	
62	53	41	31	15	29	27	26	25	21	1	2	3	4	11	12	10	7	5	6	6	
9	8	17436.05		15.43																	
Node21	21	20	19	16	18	22	23	24	17	14	28	13	30	32	33	34	35	36	39	38	
37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	61	
60	62	53	41	31	15	29	27	26	25	1	2	3	4	11	12	10	7	5	6	6	
9	8	17028.99		15.80																	
Node22	22	23	24	17	14	28	13	30	32	33	34	35	36	39	38	37	40	42	43	44	
45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	31	31	
15	29	27	26	25	21	20	19	16	18	1	2	3	4	11	12	10	7	5	6	6	
9	8	17244.85		15.60																	
Node23	23	22	18	16	19	20	21	25	26	27	29	15	17	14	28	13	30	32	33	34	
35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	57	
59	58	56	61	60	62	53	41	31	24	1	2	3	4	11	12	10	7	5	6	6	
9	8	17072.81		15.76																	
Node24	24	23	22	18	16	19	20	21	25	26	27	29	15	17	14	28	13	30	32	33	
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	55	
57	59	58	56	61	60	62	53	41	31	1	2	3	4	11	12	10	7	5	6	6	
9	8	16526.14		16.28																	
Node25	25	26	27	29	15	16	19	20	21	18	22	23	24	17	14	28	13	30	32	33	
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	55	
57	59	58	56	61	60	62	53	41	31	1	2	3	4	11	12	10	7	5	6	6	
9	8	16404.06		16.40																	
Node26	26	25	21	20	19	16	18	22	23	24	17	14	28	13	30	32	33	34	35	36	
39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	58	
56	61	60	62	53	41	31	15	29	27	1	2	3	4	11	12	10	7	5	6	6	
9	8	16638.73		16.17																	
Node27	27	29	15	16	19	20	21	25	26	24	23	22	18	17	14	28	13	30	32	33	
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	55	
57	59	58	56	61	60	62	53	41	31	1	2	3	4	11	12	10	7	5	6	6	
9	8	16271.32		16.53																	
Node28	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	30	13	31	32	
33	34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	54	
55	57	59	58	56	61	60	62	53	41	1	2	3	4	11	12	10	7	5	6	6	
9	8	17016.60		15.81																	
Node29	29	27	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	30	33	34
35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	57	
59	58	56	61	60	62	53	41	31	15	1	2	3	4	11	12	10	7	5	6	6	
9	8	16841.47		15.97																	
Node30	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	1	2	
3	4	11	12	10	7	5	6	9	8	39	36	34	33	35	32	31	41	53	62	62	
60	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40	40	
38	37	18379.90		14.64																	
Node31	31	32	33	34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	
52	54	55	57	59	58	56	61	60	62	53	41	30	13	28	14	17	24	23	22	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	6	
9	8	16621.35		16.18																	
Node32	32	33	34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	
54	55	57	59	58	56	61	60	62	53	41	31	30	13	28	14	17	24	23	22	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	6	
9	8	16720.69		16.09																	
Node33	33	34	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	
27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	39	36	37	38	40	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	62	
53	41	17368.45		15.49																	
Node34	34	35	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	
34	35	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	55	
27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	39	36	37	38	40	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	62	
53	41	17370.15		15.49																	
Node35	35	34	33	32	31	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	
27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	39	36	37	38	40	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	62	
53	41	17289.31		15.56																	
Node36	36	39	38	37	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	
58	56	61	60	62	53	41	31	32	33	34	35	30	13	28	14	17	24	23	22	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	6	
9	8	16733.98		16.07																	
Node37	37	38	40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	58	
61	60	62	53	41	39	36	34	33	35	32	31	30	13	28	14	17	24	23	22	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	6	
9	8	15966.36		16.85																	
Node38	38	37	36	39	41	31	32	33	34	35	44	43	42	40	30	13	28	14	17	24	
23	22	18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	6	
5	6	9	8	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	
62	53	18886.34		14.24																	
Node39	39	36	34	33	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	21	
25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	37	38	40	40	
42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	62	
53	41	17357.48		15.50																	
Node40	40	38	37	36	39	41	31	32	33	34	35	44	43	42	45	46	47	48	49	50	
51	52	54	55	57	59	58	56	61	60	62	53	13	28	14	17	24	23	22	18	18	
16	19	20	21	25	26	27	29	15	30	1	2	3	4	11	12	10	7	5	6	6	
9	8	18118.92		14.85																	
Node41	41	39	36	34	33	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	
21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	37	38	38	
40	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	60	
62	53	16453.44		16.35																	
Node42	42	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	
53	41	39	36	34	33	35	32	31	30	13	28	14	17	24	23	22	18	16	19	19	
20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	37	37	
38	40	17835.35		15.08																	
Node43	43	44	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	
41	39	36	34	33	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	20	
21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	37	38	38	
40	42	17928.00		15.00																	

Node44	44	43	42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24
23	22	18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	
5	6	9	8	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	
62	53	17966.79		14.97																
Node45	45	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	41	39
36	34	33	35	32	31	30	13	28	14	17	24	23	22	18	16	19	20	21	25	
26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	37	38	40	42	
43	44	17589.95		15.29																
Node46	46	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	45	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	15804.01		17.02																
Node47	47	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	45	46	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	16408.37		16.39																
Node48	48	49	50	51	52	54	55	57	59	58	56	61	60	62	53	45	46	47	43	44
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	16698.63		16.11																
Node49	49	48	47	46	45	44	43	42	40	38	37	36	39	41	31	32	33	34	35	30
13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	1	2	3	
4	11	12	10	7	5	6	9	8	50	51	52	54	55	57	59	58	56	61	60	
62	53	19275.53		13.96																
Node50	50	51	52	54	55	57	59	58	56	61	60	62	53	45	46	47	48	49	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	16738.78		16.07																
Node51	51	52	54	55	57	59	58	56	61	60	62	53	45	46	47	48	49	50	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	16916.80		15.90																
Node52	52	51	50	49	48	47	46	45	44	43	42	40	38	37	36	39	41	31	32	33
34	35	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	15	
1	2	3	4	11	12	10	7	5	6	9	8	53	62	60	61	56	58	59	57	
55	54	18876.29		14.25																
Node53	53	62	60	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	15681.65		17.15																
Node54	54	52	51	50	49	48	47	46	45	44	43	42	40	38	37	36	39	41	31	32
33	34	35	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	27	29	
15	1	2	3	4	11	12	10	7	5	6	9	8	53	62	60	61	56	58	59	
57	55	18681.59		14.40																
Node55	55	57	59	58	56	61	60	62	53	54	52	51	50	49	48	47	46	45	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	
9	8	15684.35		17.15																
Node56	56	61	60	62	53	54	52	51	50	49	48	47	46	45	44	43	42	40	38	37
36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	19	20	
21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	55	57	
59	58	19033.92		14.13																
Node57	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40	38	37	36	39	41
31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	19	20	21	25	26	
27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	53	62	60	61	56	
58	59	18865.11		14.26																
Node58	58	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40	38	37	36	39
38	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	19	20	21	
25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	53	62	60	
61	56	18374.36		14.64																
Node59	59	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40	38	37	36	39
41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	19	20	21	25	
26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	53	62	60	61	
56	58	18707.66		14.38																
Node60	60	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40
38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	
19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	
53	62	18706.87		14.38																
Node61	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43	42	40	38
37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	16	19	
20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	8	53	
62	60	18856.94		14.26																
Node62	62	60	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43	42
40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18	
16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9	
8	53	18284.60		14.71																
<-----MINIMUM PATH----->																				
NODE																				
	NODES																			
Node53	53	62	60	61	56	58	59	57	55	54	52	51	50	49	48	47	46	45	44	43
42	40	38	37	36	39	41	31	32	33	34	35	30	13	28	14	17	24	23	22	18
18	16	19	20	21	25	26	27	29	15	1	2	3	4	11	12	10	7	5	6	9
9	8	15681.65		17.15																

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