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An Optimal Ordering Policy for a Probabilistic Fixed Lifetime Inventory Model with Continuous Demand Rate

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Abstract

This paper deals with the problem of computing optimal ordering policies for the probabilistic fixed lifetime inventory model with continuous demand rate. We proposed a probabilistic fixed lifetime inventory model with continuous demand rate. The necessary condition for minimizing the expected proposed cost model was derived. The condition is also sufficient because the model is convex in S. The optimal ordering policies for this probabilistic fixed lifetime inventory system with continuous demand rate were given. The objective of the study is to examine decisions regarding when to order or not. This was investigated under some conditions. The operating characteristics obtained in this article are very significant because, for practical problems, available mathematically optimal solutions to the fixed lifetime inventory problem cannot be realized due to their computational complexity arising from the fact that exact formulation of the problem requires information on the age distribution of the items in inventory and the corresponding quantity of items of each age. Hence there is a gap between theoretical results and practical requirements for computational results. We have been able to bridge the gap between theoretical results and practical requirements for computational results. We computed the ordering cost, expected holding cost, expected shortage cost and expected outdates cost, and these computations were applied to determine the expected cost for the fixed lifetime inventory system. The expected cost model with Set- up Cost and without Set- up Cost were useful costs for the determination of the optimal ordering policy for this type of inventory system Finally, organization may like to operate under a given aspiration scenario, values of inventory level that satisfy such condition are identified and are used in the cost function to determine optimal operating conditions, this will go a long way to reduce waste and holding cost.

Keywords: Fixed lifetime; optimal ordering policies; ordering cost; expected holding cost; expected shortage cost; expected outdate cost; unique minimum.

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1 Introduction

In this paper we set up a model denoted by $E\{\hat{C}(S)\}$ for the analysis of the problem of ordering fixed lifetime inventories that deal with the characteristics of this type of problems. The lifetime inventory system consist of an age-wise profile of item (state space).The size of the space is directly proportional to the life of the items. Consequently, the inventory management is faced with lots of challenges otherwise item will be outdated. The objective of the study is to examine decisions regarding when to order or not given some conditions. Fixed lifetime products have deterministic shelf life i.e. if a product remains unused up to its lifetime; it is considered to be out-dated and must be disposed off. Important examples of items in real life include: human blood used for transfusion, food stuffs, photographic film, vaccine, batteries chemicals and other pharmaceutical products. The fixed lifetime products in inventory are usually depleted following either First-In-First-Out (FIFO) or Last-In-First-Out (LIFO) issuing policy. It is, therefore, an important problem of finding the optimal ordering policy which is closely related to that of finding suitable issuing policy for such a perishable inventory system. Inventory models for fixed lifetime perishable products have been studied by various researchers.

2 Literature Review

Products with fixed lifetime and deterministic demand, ordering policies are similar to that of non-perishable products. In these cases, one would always order in such a way that the product never perishes. If a fixed lead time is included, then one simply adjusts the reorder point so that the product does not perish. The difficulty arises when the demand is random and one has to determine the right amount of inventory to hold. The most commonly used product that has a fixed lifetime and random demand is probably the newspaper. The newsvendor problem is one of the earliest works dealing with fixed life perishability. The simple newsvendor problem deals with finding the optimal single-period order quantity for a newsvendor who faces an uncertain demand.

Firstly [1], examined the fixed lifetime perishable inventory system and obtained an estimator for the probability that an item will be sold in a given period; the probability is used to derive outdate and shortage quantities. The earliest work with respect to identifying the optimal order quantity was carried out by Ford Whitman Harris. The model developed by Harris is commonly referred to as the Economic Order Quantity (EOQ) model, and is also known as the lot sized model. The objective of the model is to find the right quantity of products to order, given the constant demand rate, the costs associated with ordering and holding inventory, such that the annual operating cost is minimized. Models have been developed for random demands with and without lead times. Performance measures like service levels, fill rates, etc. are used to calculate the safety stock to be carried and to determine order-up-to-levels and reorder points. Models were also developed to identify optimal order quantities for multi-echelon systems and systems with time-varying demand. For product with fixed lifetime and deterministic demand ordering policies are similar to that of non-perishable products. In these cases, one would always order in such a way that product never perishes. If a fixed lead time is included, then on simply adjusts the reorder point so that the product does not perish. The difficulty arises when the demand is random and one has to determine the right amount of inventory to hold. The most commonly used product that has a fixed lifetime and random demand is probably the newspaper.

Enagbonma and Eraikhuemen [2], constructed a model from which optimal ordering policies for inventory that perishes after a fixed number of periods for the discrete demand scenario. They determine ordering decisions that takes into account the perishable nature of the inventory and recommend that the first in, first out, (FIFO) optimal issuing policy for the management of stock items with fixed lifetime should be used to minimizes expected outdates. Hence, advised decision-maker to enforce the FIFO issuing policy by exposing consumers to products of the same age.

Venkata [3], states that products with a fixed lifetime are products whose lifetime is known in advance. These products do not deteriorate over the period of their lifetime but are not fit for consumption after expiry. The utility of the product is assumed to be constant till it expires. Some examples of these kinds of products are canned goods, bottled milk, processed food, pharmaceutical products, etc. while products with a random lifetime are products that can be classified as age-dependent inventory as they experience continuous deterioration. Such products can fail at any point of time. In this case, the utility of the commodity decreases with time, and it becomes essential that the product be sold as soon as possible. Examples of products of this type are fresh vegetables, fruits, and other groceries, as well as blood in blood banks and organs for organ transplants.

One of the most famous cases of perishable inventory is the classic newsvendor problem. This is a case of perishable inventory with a fixed lifetime, as a newspaper becomes obsolete after a day. The newsvendor has to make a decision on how much to order for the next day to maximize his profit. It is a one-period, stochastic demand inventory problem that has been widely studied. Azadivar and Rangarajan [4], discuss the basic problem, formulation, and the solution methodology.

Furthermore [5], investigated an optimal procurement policy for items with an inventory level dependent demand rate and fixed lifetime. A mathematical model and solution methodology were developed. [6], considered a discrete-time supply chain for perishable goods where there are separate demand streams for items of different ages. The effect of substitution was particularly considered. The substitution options given were compared analytically in terms of the infinite horizon expected costs. Results with numerical experiments are given to explore the effects of problem parameters on performance.

Agbadudu and Enagbonma [7], compared two expected cost functions in the management of fixed lifetime inventory model. since expected costs obtained in the proposed model are less than the expected cost of the single period inventory model. The models were developed under the assumption that demand has a discrete with a probability mass function (pmf) and under the deterministic fixed lifetime inventory system scenario. An extension of the study was suggested for fixed lifetime models when demand has a continuous probability density function.

Dan et al. [8], considered a solution to lifetime buy quantity optimization implemented within the Life of Type Evaluation (LOTE) tool. It has also shown LOTE’s capability to analyze complex, multi-part systems with refresh dates, changing demand profiles, and modified demand distributions. The results for the Motorola Infrastructure Base Station case indicate that demand distribution plays an important role in the results obtained. The LOTE results have also revealed that organizations making lifetime buys may be placing more emphasis on the short-term under-buy penalty costs and less on the inventory and procurement costs that contribute as much or more to the lifecycle cost, and as a result organizations may be consistently overbuying their lifetime buys.

John et al. [9], constructed an inventory model for deteriorating items with constant rate of replenishment, time and selling price dependent demand and decay has been developed and analyzed in the light of various parameters and costs and with the objective of maximizing the total system profit. The model was illustrated with numerical examples and sensitivity analysis of the model with respect to costs and parameters was also carried out. This model also includes the exponential decay model as a particular case for specific values of the parameters. The proposed model can further be enriched by incorporating salvage of deteriorated units, inflation, quantity discount, and trade credits etc. They develop and analyze an inventory model for deteriorating items with Weibull rate of decay having finite rate of replenishment and selling price dependent demand. The inventory level at any time “t” is governed by differential equations. With suitable cost considerations the total cost function and profit rate function are also obtained by maximizing the profit rate function, the optimal ordering and pricing policies of the model are derived. Nandakumar and Morton [10], derived near myopic bounds on the order quantities and used the bounds to evaluate the performance of the resulting heuristics. The near myopic approach, basically, casts any periodic inventory problem in the framework of a newsboy problem and attempts to bound the various newsboy parameters. The upper and lower bounds of various parameters lead to bound on the order quantity.

Liu and Lian [11], analyzed an (s, S) continuous review perishable inventory system with a general renewal demand process and instantaneous replenishments. Using a Markov renewal approach, they obtained closed-form solutions for the steady state probability distribution of the inventory level and system performance measures. They also constructed a closed-form expected cost function and showed that for any fixed S, the cost function is either monotone or convex in s. Kalpakam and Sapna [12], studied an (s; S) model with Poisson demand, assuming an exponential lead time and an exponential lifetime. Based on Markov chain technique, the exact cost function was obtained. Some extensions of this model have been considered.

Kalpakam and Shanthi [13], Proposed a similar model in which orders are placed only at demand epochs. Later, the authors consider the case of renewal demand [14], [15] considered a similar model and derived the total expected cost function. Perry [16], considered a perishable inventory system where the commodity's arrival and customer demand processes are stochastic and the stored items have a constant lifetime. The stock level is represented by the amount arriving during the life of the oldest item and it is assumed to fluctuate as an alternating two-sided regulated Brownian motion between barriers 0 and 1. Hitting of level 0 are outdating and hitting of level 1 are unsatisfied demands. A useful martingale is introduced for analyzing the controlled process as well as the total expected discounted cost. Chiu [17], developed an approximate expression for the total expected average cost per unit time for the fixed lifetime inventory system under the assumption of positive lead time, but could not prove that the cost function was convex; however an iterative scheme for solving the problem was given.

Williams and Patuwo [18], also obtained optimal order quantities for a product with a two- period lifetime and a positive lead time for replenishment. Most of the fixed lifetime models and all of the aforementioned models are based on the periodic review policy. Chiu [19], later developed a good approximation for the expected inventory level per unit time and compared it with existing approaches. The author also showed that inappropriate approximations result in distortions when identifying optimal policies.

Goh et al. [20], studied perishable inventory models with random replenishments (with blood donation processes in mind). They have established the equivalences of the time between successive outdating and the time between successive demand losses with the busy periods of the corresponding single server queues with impatient customers.

Ravichandran [21], analyzed a perishable model with a positive random lead time and a Poisson demand process. By assuming that the aging of the new stock only begins after the complete depletion of the existing stocks, some analytical results were obtained.

3 Assumptions of the Proposed Model

The proposed model is developed under the following assumptions.

(i) The issuing policy is first in, first out (FIFO).
(ii) Periodic review and order-up to order policy with parameter S is used.
(iii) Units expire after the age m periods in the inventory system.
(iv) Time is divided into discrete periods.
(v) The length of a period is arbitrary but fixed.
(vi) The lead time is zero.
(vii) Demands in successive periods are independent and identically distributed random variables with known distribution.
(viii) The sequence of events within each period is as follows: (a) an order is placed and order arrives immediately,(b) demand for the period is filled and (c) any unit that has reached the age m and has not been used is outdated.
(ix) he order quantity is determined as follows. If IP is the inventory position at the time of placing order, the order quantity is $Q = S - IP$.

3.1 Mathematical Notations

The following Mathematical notations were used in the development of the proposed model.

K = setup cost

c = replenishment cost

θ = mean demand

S = Inventory level at the start of each period

h = holding cost

p = shortage cost

i = amount demanded

f(t) = probability density function of the uniform distribution

m = fixed life time

D(t)= demand in period t

E(.) = the expectation function

E (D(t)) = θ

$E\{C(S)\} =$ Expected total relevant cost function for the period without Setup cost.

$E\{\hat{C}\,(S)\} =$ Expected total relevant cost function for the period with Setup cost

E = Total expected average cost per unit time for the fixed lifetime inventory system under the assumption of positive lead time

$\hat{C}$ = Average cost per unit time

IP = Inventory Position

W = Expected outdate quantity

Q = Expected ordered quantity

The model is based on the probability that an item in the inventory system is sold in a period. The estimator for the probability that an item will be sold is given in [1] as

$$p = \frac{\theta}{S} \tag{1}$$

Let Q be the expectation of units ordered at the end of period t and received

at the beginning of period t + 1, given by the same expression of [1]

$$Q = E\left(Q(t)\right) = \frac{\theta}{\left(1-\left(1-\frac{\theta}{S}\right)^{m}\right)} \tag{2}$$

and W the expectation of units outdated at the end of period t also from [1] given by

$$W = E\left(W(t)\right) = Q\left(1 - \frac{\Theta}{S}\right)^m \tag{3}$$

Succinctly 1 – P is the probability that an item is not sold in one period. $(1 - P)^m$ is the probability that an item will be outdated, this is based on the fact that transaction in the period S are independent.

In particular outdate decreases with increasing m, since 1 – P < 1. This result was conjecture by Nahmias (1977). Using (2) in (3) we obtain

$$w = \frac{\theta(1-P)^m}{1-(1-P)^m} \tag{4}$$

This is an expression similar to that given by [22] and the same equation of [2] for the daily outdates quantity of cross-match blood, P is the proportion of cross-match blood that is actually transfused.

Where D (t) is demand in period t,

$$E\left(D(t)\right) = \theta \tag{5}$$

$$\text{Ordering cost} = c(S - IP) \tag{6}$$

$$\text{Expected holding cost} = h\int_0^s (S - t)\, f(t)\, dt \tag{7}$$

The probability of running out of stock is given by

$$P_{out} = \int_S^\infty f(t)\, dt = 1 - \int_0^S f(t)\, dt \tag{8}$$

and the shortage quantity is given by

$$Z = \int_S^\infty (t - S)\, f(t)\, dt \tag{9}$$

$$\text{Expected shortage cost} = p\int_S^\infty (t - S)\, f(t)\, dt = p[\theta - \int_0^S t\, f(t)\, dt - S\, P_{out}] \tag{10}$$

Applying (8) in (10) yields

$$\text{Expected shortage cost} = p\int_S^\infty (t - S)\, f(t)\, dt = p[\theta - \int_0^S t\, f(t)\, dt - S(1 - \int_0^S f(t)\, dt)] \tag{11}$$

$$\text{Expected outdate cost} = w\int_0^s (S - t)\, f(t) dt \tag{12}$$

Applying (6), (.7), (10) and (12) yields the mathematical model given by

$$E\{\hat{C}\,(S)\} = K + c\,(S - IP) + h\int_0^S (S - t)\, f(t)\, dt + p\int_S^\infty (t - S)\, f(t)\, dt + w\int_0^S (S - t)\, f(t) dt \tag{13}$$

3.2 The Necessary Condition for a Minimum

The function can be shown to be convex in s, thus having a unique minimization. Taking the first derivative of $E\{\hat{C}\,(S)\}$ with respect to s and equating to zero,

$$E\{\hat{C}'\,(S)\} = c + h\int_0^s f(t) dt - p\int_0^\infty f(t) dt + w\int_0^s f(t) dt = 0$$

$$c + h\, P\{t \le S\} - p(1 - P\{t \le S\} + w\, P\{t \le S\}) = 0$$

$$c + h P\{t \le S\} - p + pP\{t \le S\}) + w P\{t \le S\} = 0$$

$$h P\{t \le S\} + w P\{t \le S\} + p P\{t \le S\} = p - c$$

$$P\{t \le S\} (h + p + w) = p - c$$

$$P\{t \le S^*\} = \frac{p - c}{h + p + w}$$

The condition is also sufficient since $E\{\hat{C}(S)\}$ is a convex function

Since K is the setup cost assumed to be independent of the quantity ordered or produced, the minimum value of $E\{\hat{C}(S)\}$ must occur at S* ,hence $E\{C(S)\}$ and $E\{\hat{C}(S)\}$ must appear as shown in fig .2

4 Numerical Example and Guidelines

We give an example to illustrate the method and suggest guidelines for the management of the fixed lifetime inventory problem. We considered inventory system with a probability density function of the uniform distribution .The ordered and outdate quantities are computed using the equation (2) and the equation (4) for (say) fixed lifetime m = 4, and mean θ = 35, inventory level S = 37, 38, 39,…, 347 the results are given in tables 1 and 2 and these quantities are applied in the expected cost function. The results are given in tables 3, 4, 5, and 6.

Table 1. Operating characteristics (S = 37 to 62)

Inventory level (S)	Expected ordered quantity Q = E(Q(t))	Inventory position	Expected outdate quantity W = E(W(t))
37	35.0003	1.9986	0.0003
38	35.0014	2.9961	0.0014
39	35.0039	3.9915	0.0039
40	35.0085	4.9839	0.0085
41	35.0161	5.9730	0.0161
42	35.0270	6.9580	0.0270
43	35.0420	7.9386	0.0420
44	35.0614	8.9144	0.0614
45	35.0856	9.8852	0.0856
46	35.1148	10.8506	0.1148
47	35.1494	11.8107	0.1494
48	35.1893	12.7652	0.1893
49	35.2348	13.7142	0.2348
50	35.2858	14.6576	0.2858
51	35.3424	15.5956	0.3424
52	35.4044	16.5281	0.4044
53	35.4719	17.4552	0.4719
54	35.5448	18.3771	0.5448
55	35.6229	19.2939	0.6229
56	35.7061	20.2057	0.7061
57	35.7943	21.1126	0.7943
58	35.8874	22.0147	0.8874
59	35.9853	22.9123	0.9853
60	36.0877	23.8054	1.0877
61	36.1946	24.6942	1.1946
62	36.3058	25.5789	1.3058

Fig. 1, Indicates expected costs against inventory level. Specifically, a positive directional relationship exist between (a) ordering cost and inventory level, (b) expected holding cost and inventory level, (c) expected outdate cost and inventory level. However an inverse relationship between expected shortage cost and inventory level, Fig. 2, indicates (y-Y) optimal ordering policies. Fig. 2 was obtained by plotting $E\{\hat{C}(S)\}$ and $E\{C(S)\}$ on the scale. The Tables 1-2 indicates expected ordered quantities, inventory positions and expected outdate quantities when $m = 4$, $\theta = 35$ and inventory level $s = 37, 38, 39 \ldots 347$.

Tables 3 – 6 indicates set up cost, ordering cost, expected holding cost, expected shortage cost , expected outdates cost, expected cost without setup cost $\{EC(S)\}$ and expected cost with setup cost $\{E\hat{C}(S)\}$.

The value Y is equal to S* while the value y is determined from

$E\{C(y)\} = E\{\hat{C}(Y)\} = K + E\{C(Y)\}$, Such that $y < Y$.

If $IP < y$, order $Y - IP$

If $IP \geq y$, do not order.

From the analysis using the proposed method, we recommend the following guideline for the management of the fixed lifetime inventory system. Firstly, the model is based on average demand, the demand distribution and inventory level. Secondly, effort should be geared towards obtaining accurate information on the demand distribution.

Table 2. Operating characteristics (S = 63 to 347)

Inventory level (S)	Expected ordered quantity Q = E(Q(t))	Inventory position	Expected outdate quantity W = E(W(t))
63	36.4211	26.4596	1.4211
64	36.5404	27.3363	1.5404
65	36.6637	28.2094	1.6637
66	36.7906	29.0788	1.7906
67	36.9212	29.9447	1.9212
68	37.0553	30.8073	2.0553
69	37.1927	31.6667	2.1927
70	37.3333	32.5229	2.3333
71	37.4771	33.3761	2.4771
72	37.6239	34.2265	2.6239
73	37.7735	35.0740	2.7735
74	37.9260	35.9189	2.9260
75	38.0811	36.7612	3.0811
76	38.2388	37.6010	3.2388
77	38.3990	38.4384	3.3990
78	38.5616	39.2734	3.5616
79	38.7266	40.1062	3.7266
80	38.8938	40.9369	3.8938
81	39.0631	41.7655	4.0631
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
342	99.8020	241.9516	64.8020
343	100.0484	242.7052	65.0484
344	100.2948	243.4588	65.2948
345	100.5412	244.2123	65.5412
346	100.7877	244.9658	65.7877
347	101.0342	347.0000	66.0342

Table 3. Operating characteristics (S = 37 to 52)

Inventory level (s)	Setup cost	Ordering cost	Expected holding cost	Expected shortage cost	Expected outdate cost	{EC(S)}	{EĈ(S)}
37	50	4200.0359	462.8378	2475	0.0055	7137.8792	7187.8792
38	50	4200.1632	475.0000	2400	0.0258	7075.1890	7125.1890
39	50	4200.4648	487.8205	2325	0.0755	7013.3609	7063.3609
40	50	4201.0256	500.0000	2250	0.1709	6951.1966	7001.1966
41	50	4201.9272	512.8049	2175	0.3292	6890.0613	6940.0613
42	50	4203.2432	525.0000	2100	0.5676	6828.8108	6878.8108
43	50	4205.0380	537.7907	2025	0.9026	6768.7313	6818.7313
44	50	4207.3649	550.0000	1950	1.3502	6708.7152	6758.7152
45	50	4210.2674	562.7778	1875	1.9251	6649.9703	6699.9703
46	50	4213.7788	575.0000	1800	2.6409	6591.4197	6641.4197
47	50	4217.9239	587.7660	1725	3.5101	6534.1999	6584.1999
48	50	4222.7196	600.0000	1650	4.5439	6477.2635	6527.2635
49	50	4228.1761	612.7551	1575	5.7526	6421.6838	6471.6838
50	50	4234.2978	625.0000	1500	7.1454	6366.4432	6416.4432
51	50	4241.0843	637.7451	1425	8.7304	6312.5598	6362.5598
52	50	4248.5312	650.0000	1350	10.5151	6259.0463	6309.0463

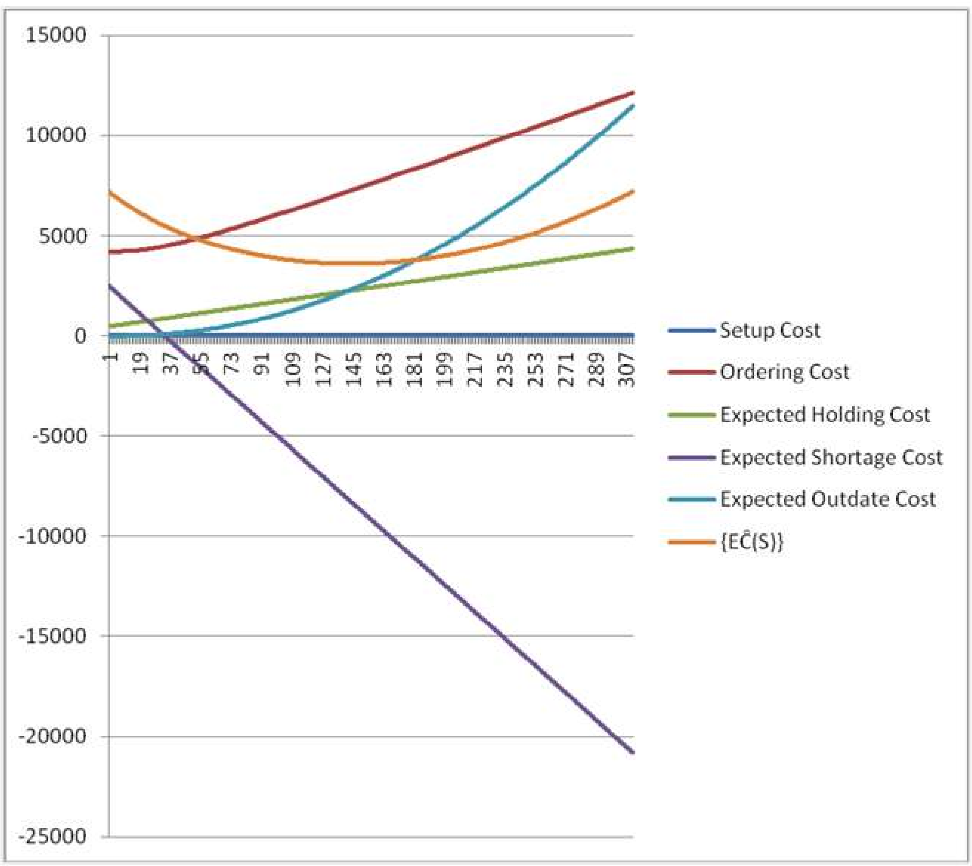


Fig. 1. Expected Costs against the Inventory Level

Table 4. Operating characteristics (S = 179 to 196)

Inventory level (s)	Setup cost	Ordering cost	Expected holding cost	Expected shortage cost	Expected outdate cost	{EC(S)}	{$E\hat{C}$(S)}
179	50	7226.8019	2237.5698	-8175	2257.4897	3546.8614	3596.8614
180	50	7255.0995	2250.0000	-8250	2291.3246	3546.4242	3596.4242
181	50	7283.4175	2262.5691	-8325	2325.4107	3546.3972	3596.3972
182	50	7311.7553	2275.0000	-8400	2359.7477	3546.5030	3596.5030
183	50	7340.1126	2287.5683	-8475	2394.3359	3547.0168	3597.0168
184	50	7368.4892	2300.0000	-8550	2429.1751	3547.6643	3597.6643
185	50	7396.8847	2312.5676	-8625	2464.2653	3548.7175	3598.7175
186	50	7425.2987	2325.0000	-8700	2499.6065	3549.9052	3599.9052
187	50	7453.7309	2337.5668	-8775	2535.1987	3551.4965	3601.4965
188	50	7482.1811	2350.0000	-8850	2571.0419	3553.2230	3603.2230
189	50	7510.6489	2362.5661	-8925	2607.1360	3555.3511	3605.3511
190	50	7539.1341	2375.0000	-9000	2643.4811	3557.6152	3607.6152
191	50	7567.6362	2387.5654	-9075	2680.0772	3560.2788	3610.2788
192	50	7596.1551	2400.0000	-9150	2716.9241	3563.0792	3613.0792
193	50	7624.6905	2412.5648	-9225	2754.0220	3566.2773	3616.2773
194	50	7653.2421	2425.0000	-9300	2791.3707	3569.6129	3619.6129
195	50	7681.8097	2437.5641	-9375	2828.9704	3573.3441	3623.3441
196	50	7710.3929	2450.0000	-9450	2866.8209	3577.2138	3627.2138

Table 5 . Operating characteristics (S = 53 to 185)

Inventory level (s)	Setup cost	Ordering cost	Expected holding cost	Expected shortage cost	Expected outdate cost	{EC(S)}	{$E\hat{C}$(S)}
53	50	4256.6308	662.7358	1275	12.5060	6206.8726	6256.8726
54	50	4265.3727	675.0000	1200	14.7089	6155.0815	6205.0815
55	50	4274.7445	687.7273	1125	17.1290	6104.6008	6154.6008
56	50	4284.7323	700.0000	1050	19.7709	6054.5031	6104.5031
57	50	4295.3206	712.7193	975	22.6386	6005.6786	6055.6786
58	50	4306.4935	725.0000	900	25.7359	5957.2294	6007.2294
59	50	4318.2342	737.7119	825	29.0659	5910.0120	5960.0120
60	50	4330.5256	750.0000	750	32.6314	5863.1570	5913.1570
61	50	4343.3504	762.7049	675	36.4349	5817.4902	5867.4902
62	50	4356.6914	775.0000	600	40.4786	5772.1700	5822.1700
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
179	50	7226.8019	2237.5698	-8175	2257.4897	3546.8614	3596.8614
180	50	7255.0995	2250.0000	-8250	2291.3246	3546.4242	3596.4242
181	50	7283.4175	2262.5691	-8325	2325.4107	3546.3972	3596.3972
182	50	7311.7553	2275.0000	-8400	2359.7477	3546.5030	3596.5030
183	50	7340.1126	2287.5683	-8475	2394.3359	3547.0168	3597.0168
184	50	7368.4892	2300.0000	-8550	2429.1751	3547.6643	3597.6643
185	50	7396.8847	2312.5676	-8625	2464.2653	3548.7175	3598.7175

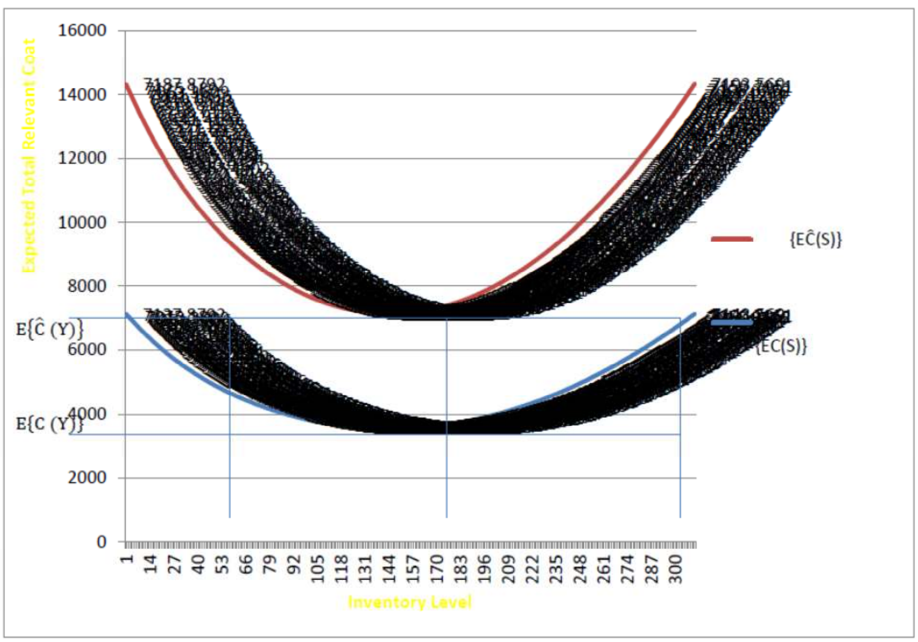
4.1 No-Setup Cost Model

The model without the setup cost is given by the equation (14)

$$E\{C(S)\} = c(S - IP) + h\int_0^S (S-t)\, f(t)\, dt + p\int_S^\infty (t-S)\, f(t)\, dt + w\int_0^S (S-t)\, f(t)dt \tag{14}$$

Table 6. Operating Characteristics (S = 186 to 347)

Inventory level (s)	Setup cost	Ordering cost	Expected holding cost	Expected shortage cost	Expected outdate cost	{EC(S)}	{EĈ(S)}
186	50	7425.2987	2325.0000	-8700	2499.6065	3549.9052	3599.9052
187	50	7453.7309	2337.5668	-8775	2535.1987	3551.4965	3601.4965
188	50	7482.1811	2350.0000	-8850	2571.0419	3553.2230	3603.2230
189	50	7510.6489	2362.5661	-8925	2607.1360	3555.3511	3605.3511
190	50	7539.1341	2375.0000	-9000	2643.4811	3557.6152	3607.6152
191	50	7567.6362	2387.5654	-9075	2680.0772	3560.2788	3610.2788
192	50	7596.1551	2400.0000	-9150	2716.9241	3563.0792	3613.0792
193	50	7624.6905	2412.5648	-9225	2754.0220	3566.2773	3616.2773
194	50	7653.2421	2425.0000	-9300	2791.3707	3569.6129	3619.6129
195	50	7681.8097	2437.5641	-9375	2828.9704	3573.3441	3623.3441
196	50	7710.3929	2450.0000	-9450	2866.8209	3577.2138	3627.2138
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
341	50	11946.6701	4262.5367	-20325	11006.7271	6890.9338	6940.9338
342	50	11976.2355	4275.0000	-20400	11081.1356	6932.3711	6982.3711
343	50	12005.8036	4287.5364	-20475	11155.7943	6974.1344	7024.1344
344	50	12035.3744	4300.0000	-20550	11230.7032	7016.0776	7066.0776
345	50	12064.9477	4312.5362	-20625	11305.8623	7058.3462	7108.3462
346	50	12094.5236	4325.0000	-20700	11381.2715	7100.7951	7150.7951
347	50	12124.1021	4337.5360	-20775	11456.9309	7143.5690	7193.5690



$y \longrightarrow Y \longrightarrow y_1$

Order Do not order

Fig. 2. (y-Y) Optimal Ordering Policy

4.2 Setup Cost Model

The model with setup cost K is given by the equation (15)

$$E\{\hat{C}\,(S)\} = K + c(S - IP) + h\int_0^S (S-t)\,f(t)\,dt + p\int_S^\infty (t-S)\,f(t)\,dt + w\int_0^S (S-t)\,f(t)dt \quad (15)$$

4.3 The y-Y Ordering Policy for the Inventory System with Fixed Lifetime

The symbols y and Y are defined in fig 2. The value Y is equal to s* while the value y is determined from
$E\{C(y)\} = E\{\hat{C}\,(Y)\} = K + E\{C(Y)\}$
Such that y<Y.

Assuming IP, the amount on hand before an order is placed, the number of quantity to be is answered under these three conditions.

i $IP < y$

ii $y \le IP \le Y$

iii $IP > Y$

Condition 1: (IP<y)

Because IP is already on hand, its equivalent cost is given by $E\{C(IP)\}$. If any additional amount s-IP(s>IP) is ordered, the corresponding cost given s is $E\{\hat{C}\,(S)\}$, which includes the setup cost K. from above see fig 2 , we have:

$$\min_{s > IP} E\{\hat{C}\,(S)\} = E\{\hat{C}\,(Y)\} < E\{C(IP)\}$$

The implication is that the optimal inventory levels reach S* = Y and the amount ordered equals Y-IP

Condition 2 : ($y \le IP \le Y$)

From data extracted from Fig. 2, we have,

$E\{C(IP)\} \le \min_{s>IP} E\{\hat{C}(S)\} = E(\hat{C}(Y)$

Thus, it is not advantageous to order in this case and $S^* = IP$.

Condition 3: (IP > Y)

From data extracted from Fig. 2, we have for s > IP,

$E(C(IP) < E\{\hat{C}(S)\}$
The condition indicates that, as in condition (2), it is not advantageous to place an order- that is $s^* = IP$.

The optimal inventory policy, frequently referred to as the y-Y polic**y,** is summarized as:

If $IP < y$, order Y-IP

If $IP \ge y$, do not order.

The optimality of the y-Y policy is guaranteed because the associated cost function is convex.

5 Conclusions

We have computed optimal ordering policies for a probabilistic fixed lifetime inventory system. Ordering decisions that take into account the perishable nature of the inventory were determined. The first in, first out, (FIFO) optimal issuing policy for the management of stock items with fixed lifetime should be used. The FIFO policy minimizes expected outdates. So, it is advisable for the decision-maker to enforce this issuing policy by exposing consumers to products of the same age. Contrariwise, if the consumer enforces the issuing policy, the last in, first out (LIFO) policy will result generally. From the analysis of the proposed model we found out that If inventory position IP $< y$, order Y-IP. However, If inventory position IP $\geq y$, do not order. Important practical use of the model in real life include: human blood used for transfusion, food stuffs, photographic film, vaccine, batteries chemicals and other pharmaceutical products.

Competing Interests

Authors have declared that no competing interests exist.

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