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High-accuracy measurements on biperiodical circuits

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Abstract

Coupled resonators in an assembled structure lose their individuality and in co-operation contribute to the generation of structure modes (resonant frequencies). The resonant frequencies of these modes are the only measurable quantities. In order to predict structural behaviour in a variety of cases, the problem that arises is the extraction of all the parameters characterizing the structure from the measurements mentioned here. If all the modes are confined in a bandwidth that is small with respect to the central frequency, the total coupled resonator system is well represented by a circuit of unknown lumped constants.

The structure modes are the solutions of the equation obtained by equating to zero the determinant relevant to the lumped circuit representation. The equation is a polynomial of the squared frequency variable, the degree of which is equal to the number M of circuits.

The analysis method described in this paper consists in varying, by an unknown amount, the frequency of a single resonator in the chain. This variation will produce a change in the frequencies of all structure modes. It is possible to find certain invariants linearly dependent on all the unchanged parameters of the circuit. These invariants have an algebraic representation that allows the extraction of the structure parameter values with extremely high accuracy. The proposed method is quite general and, in the present work, we give an example applying the method to the characterization of a side-coupled linac (SCL).

Keywords: coupled circuit, modes, side-coupled linacs

(Some figures may appear in colour only in the online journal)


1. Introduction

A linear accelerator (generally named linac) is made of single cavities, each one characterized by its own resonant frequencies. When cavities are coupled together, the entire structure will exhibit modal frequencies distinct from those of each individual cavity. Because these frequencies are the only directly measurable quantities, it is important to develop a method for the extraction of the structure's parameters in order to predict its behaviour.

In the following, we will present a general methodology, based on:

- 1) step variation of the frequency relevant to one single cavity in measurements of all the system modes;
- 2) finding a certain number of appropriate algebraic combinations of the measured frequencies, which is invariant with respect to the cavity frequency variation;
- 3) relating the invariants to the parameters characterizing the structure.

In this paper, we will apply this method to a biperiodical chain of resonators, and in particular to a side-coupled linac (SCL). However, this method can be used for the characterization of different types of linear accelerators based on the

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principle of *coupling*, such as a side-coupled drift tube linac (SCDTL) [1] or a coupled cavity linac (CCL), for protons and heavy ions as well as for electrons. The method proposed here is a generalization of the one described in [1]. Because of the novelty of the method, it has not yet been applied by other researchers.

An SCL is a biperiodic system of resonators formed by a number of accelerating cavities (AC) on axis with the travelling particles coupled with a certain number of off-axis coupling cavities (CC). When the cavities are assembled, the whole system will resonate at frequencies, F_n , each one characterized by its own field phase advance from cavity to cavity. For a biperiodical system, we define a first-order constant (k_i) for adjacent cavities, and second-order constants (k_a, k_c) for coupling between AC elements and between CC elements, respectively.

If the coupling constant is sufficiently small all the modes are confined in a bandwidth that is small with respect to the central frequency. It has been already demonstrated that, in this case, a coupled cavity system is well represented by a lumped constant circuit [2–6], regardless on the type of coupling, capacitive or inductive. As a matter of fact, in our example some coupling constants are negative, which indicates a capacitive coupling. The condition on the smallness of the bandwidth is necessary to ensure that the fundamental modes do not couple with the higher-order ones, therefore invalidating the lumped circuit representation. An example relevant to five cavities is depicted in figure 1.

This representation is extremely fruitful for describe the behaviour of the cavity system since it will allow the extraction from the modal frequencies F_n of all the parameters characterizing the linac, e.g. the frequencies of each cavity. With this information, we may tune those resonators whose frequencies are outside the fixed tolerances.

An SCL for low-energy protons is a standing-wave linac, which is fed from the centre of the structure. Typically, all the cavities are longitudinally symmetrical in order to assure that the coupling coefficients between AC and CC cavities are equal, providing an almost uniform accelerating electric field along the linac.

The method developed could be used not only for linac characterization, but also for an easy optimization of the structure once some quality indices are fixed.

2. Analysis of determinant properties

In a real chain of resonators, not all the cavities and coupling constants are equal, giving rise to an asymmetric system with respect to the central cavity.

Allowing for a more general representation than the one reported in figure 1, all the frequencies and the coupling coefficients will be different. The modal frequencies, F_n , will satisfy the following equation:

$$D_M(f_m, k_i, k'_j; F_n) = 0; \quad M = 4N + 1 \quad (1)$$

$$1 \leq m \leq M; \quad 1 \leq i \leq 4N; \quad 1 \leq j \leq 4N - 1$$

where D_M is the determinant of the circuit, M is the total number of resonators, f_m are the cavity frequencies, and k_i

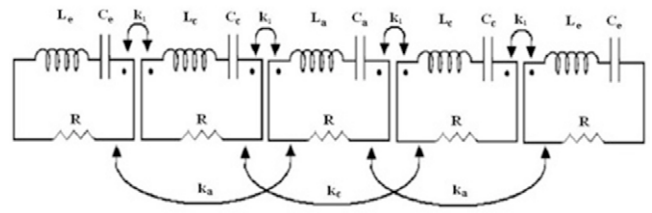


Figure 1. An example of the equivalent circuit.

and k'_j are respectively the first- and second-order coupling constants.

For each pair of cavities (in symmetric positions with respect to the central one) we can write:

$$f_m = f_m^0 \left(1 + \frac{\Delta f_m}{f_m^0} \right); \quad f_{M+1-m} = f_m^0 \left(1 - \frac{\Delta f_m}{f_m^0} \right) \quad (2)$$

where

$$f_m^0 = \frac{f_m + f_{M+1-m}}{2} \quad \text{and} \quad \Delta f_m = \frac{f_m - f_{M+1-m}}{2}$$

An analogous position will be set for the constants k_i and k'_j .

We define the polynomial D_M^0 as

$$D_M^0(f_m^0, k_i^0, k'_j{}^0; F) \equiv D_M(f_m^0, k_i^0, k'_j{}^0; F)$$

The polynomial D_M^0 is invariant with respect to the following interchanges:

$$f_m^0 \Leftrightarrow f_{M+1-m}^0, \quad k_i^0 \Leftrightarrow k_{M-i}^0, \quad k'_j{}^0 \Leftrightarrow k'_{M-1-j}{}^0$$

Because of that invariance, it can be demonstrated that the polynomial D_M is stationary around the values of symmetrized variables. Therefore, the derivative of $D_M^0(f_m^0, k_i^0, k'_j{}^0; F)$ with respect to each symmetrized parameter vanishes and the following expansion holds:

$$D_M = D_M^0 + \sum_1^M \left(\frac{\Delta f_m^0}{f_m^0} \right)^2 P_m + \sum_1^{M-1} \left(\frac{\Delta k_i^0}{k_i^0} \right)^2 Q_i \quad (3)$$

$$+ \sum_1^{M-2} \left(\frac{\Delta k'_j{}^0}{k'_j{}^0} \right)^2 R_j + \text{H.O.T.}$$

where P_m, Q_i , and R_j are polynomials functions of the symmetrized parameters only. If the asymmetries in the cavity parameters are much smaller than the nominal symmetrized values (e.g., 0.1%), as happens in most real cases, higher-order terms can be neglected. Therefore, the roots of equation (1) can be easily found by solving the following simpler equation

$$D_M^0(f_m^0, k_i^0, k'_j{}^0; F) = 0 \quad (4)$$

In addition to this we consider the lossless case (all the resistances vanishing). In this case, due to the mentioned symmetry, one can demonstrate that, for any order of polynomial, D_M^0 may be factorized as the product of two equations of lower order, namely:

$$D_M(f_m^0, k_i^0, k'_j{}^0; F) = G_{2N}(f_m^0, k_i^0, k'_j{}^0; F) \quad (5)$$

$$\times G_{2N+1}(f_m^0, k_i^0, k'_j{}^0; F)$$

where the functions G_{2N} and G_{2N+1} are polynomials of order $2N$ and $2N + 1$. As a consequence the roots can be found by solving two algebraic equations of lower degree. This property greatly simplifies the behaviour analysis of SCLs.

2.1. A particular case: $M = 5$

In order to illustrate the theory, without loss of generality we may allow for a circuit of five resonators, in which the values of the resistors are very small in order to be neglected.

The modal frequencies F_n satisfying equation (1) will be solutions of the following equation:

$$D_5^0(f_e, f_a, f_c, k_1, k_a, k_c; F) = 0 \quad (6)$$

where f_e, f_a , and f_c stand for the frequencies of *end*, *accelerating*, and *coupling* cavities. The constants k_1, k_a, k_c are, respectively, the main coupling constant between consecutive cavities, the coupling constant between ACs, and the coupling constant between CCs. We recall that those constants and the frequencies f_e, f_a , and f_c are the unknowns of the system; conversely, the mode frequencies F_n are the only measurable quantities.

By resorting to symbolic codes, in this case we may factorize equation (6). The explicit expressions of the polynomials are:

$$G_2(f_e, f_c, k_1, k_c; F_n); n = 2, 4 \quad (7)$$

$$G_3(f_e, f_a, f_c, k_1, k_a, k_c; F_n) = 0; n = 1, 3, 5 \quad (8)$$

where the label has been ordered according to the increasing value of the frequency.

It is apparent that in equation (7) the central cavity f_a and the coupling constant k_a do not appear. This means that there is one set of frequencies, which is independent of those two parameters. Its solution is a driving term of the second polynomial.

This behaviour simplifies the calculation of the other unknowns. Equations (7) and (8) are satisfied by the roots of even order and odd order, respectively.

3. The quest for invariants

Given an algebraic equation of arbitrary degree, its coefficients can be represented as combinations of their roots. This representation depends on the order of the equation and the positions of the coefficients. Allowing for equation $G_2(\dots)$, this representation gives:

$$G_2(\dots) = (F^2)^2 - (F_2^2 + F_4^2)F^2 + F_2^2F_4^2 \quad (9)$$

On the other side, we have from the factorization of equation (6):

$$G_2(\dots) = (F^2)^2 - \frac{4f_c^2 + (4 - 2k_c)f_e^2}{4 - 2k_c - k_1^2}F^2 + \frac{4f_e^2f_c^2}{4 - 2k_c - k_1^2} = 0 \quad (10)$$

By comparing equations (9) and (10) one may write the following two equalities:

$$\begin{aligned} F_2^2 + F_4^2 &= \frac{4f_c^2 + (4 - 2k_c)f_e^2}{4 - 2k_c - k_1^2} \\ F_2^2 \cdot F_4^2 &= \frac{4f_e^2f_c^2}{4 - 2k_c - k_1^2} \end{aligned} \quad (11)$$

Starting from equation (11), our method consists in extracting an unknown, e.g. f_c^2 , from one equation and inserting it in the other one. We obtain the following linear equation:

$$F_2^2 \cdot F_4^2 = f_c^2(F_2^2 + F_4^2) - \frac{f_e^4(4 - 2k_c)}{4 - 2k_c - k_1^2} \quad (12)$$

which may be shortened as:

$$y = Ax + B \quad (13)$$

where the variables x and y are defined as

$$\begin{cases} x = F_2^2 + F_4^2 \\ y = F_2^2 \cdot F_4^2 \end{cases} \quad (14)$$

Here we have carried out the procedure indicated in step 2 described in the Introduction: it is apparent that the coefficient A as well as the known term B do not change with respect to the variation of the frequency f_c^2 obtained by means of an *ad hoc* tuner. During this operation the values of y and x move along a straight line.

Therefore, we have found *two invariants*. In principle, it could be sufficient to make only two measurements relevant to two different values of f_c in order to find the values of A and B . However, in order to get the best estimate and the lowest error many measurements should be taken at different values of the frequency f_c .

Likewise, we may proceed by coupling the coefficients of equation $G_3(\dots) = 0$ among themselves and with those of equation $G_2(\dots) = 0$. We may obtain $M(M - 1)$ equations similar to equation (12); in all of them the angular coefficient A has always the dimension of f^2 .

Once the value of a certain number of invariants is found, we get a set of nonlinear algebraic equations to calculate the unknowns f_e, f_a, f_c, k_1, k_a , and k_c .

The method hereby proposed is a generalization of the one described in [1] which allows for simpler circuits characterized only by the first-order coupling.

As stressed in the abstract, the coupled resonators in an assembled structure lose their individuality and in co-operation contribute to the generation of the structure modes. One these modes is chosen for feeding the structure ($\pi/2$ mode in [3], p. 105). This frequency has to be known with a precision satisfying the following inequality:

$$\frac{\Delta f}{f} \ll \frac{1}{Q} = 1.67 \times 10^{-4} \quad (15)$$

where Q is in general of the order of 6000 for normal conducting cavities at 3 GHz. In order to satisfy the above inequality, the cavity frequencies must be measured and tuned with a precision smaller than the above value. If one neglects the higher-order coupling constants, one will never satisfy this constraint.

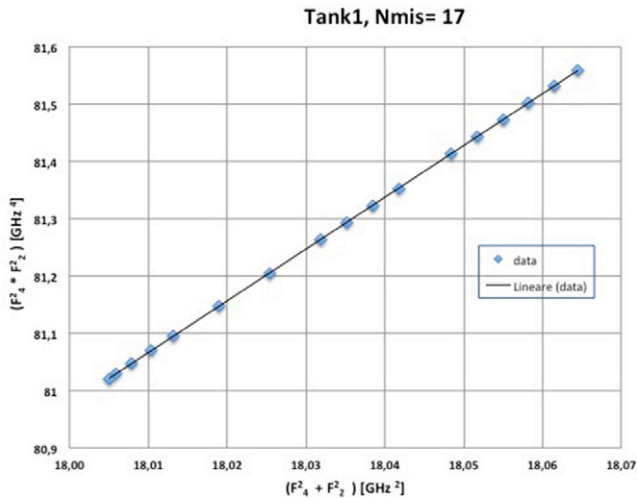


Figure 2. The straight line method according to equation (13).

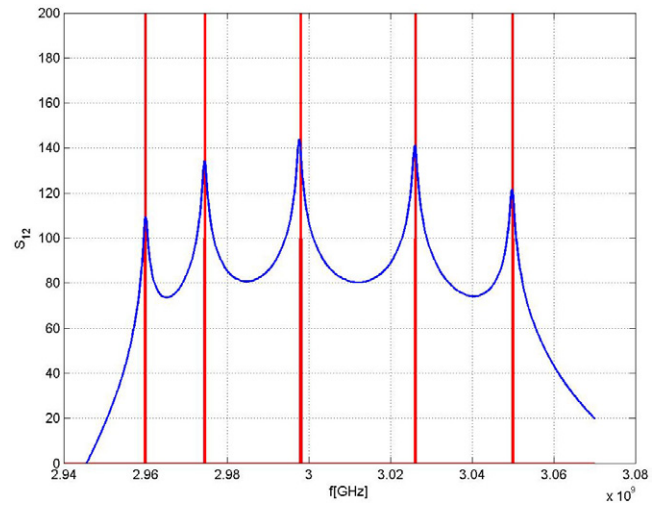


Figure 3. Comparison between measured and circuit simulation data.

4. Measurements

The first module (two tanks) of ACLIP [7] has been conceived in such a way that each cavity has two threaded frequency tuners, with 0.8 mm pitch. The maximum excursion in frequency for each tuner is about 6 MHz.

In order to measure the linac parameters with high accuracy, the analysis has been performed isolating a certain number of cavities representative of the whole tank (three ACs plus two CCs). This model can be assumed as the device under test for virtual (numerical) and real ‘measurements.’

The first goal is to equalize the symmetric cavities. To this end, the frequencies of the resonators were measured to within the same systematic error (if any). Afterwards, for different values of the frequency f_c as varied by means of the coupling cavity tuners, the five resonant mode frequencies F_m were measured.

Resorting to the equation $G_2(\dots) = 0$ and according to equations (13) and (14), the pattern shown in figure 2 is obtained using the frequencies F_2 and F_4 . It is apparent that the points match very well with a straight line.

According to equation (13), the angular coefficient gives, without intermediate steps, the value of f_e . The constant k_1 can be extracted by combining the angular coefficient and the known term of equation (13). The parameter values with their errors are reported in table 1 for the case shown in figure 2.

A similar procedure can be adopted for the factorized equation $G_3(\dots) = 0$. The same measured spectrum can be used to extract the other structure parameters, f_a, f_c , and k_a , resorting now to the frequencies F_1, F_3 , and F_5 . Upon combining the coefficients of the second- and third-order equations among themselves, we may obtain all $M(M-1)$ relations. As an example we choose the following pairs of variables:

$$\begin{cases} x = F_1^2 + F_3^2 + F_5^2 \\ y = F_1^2 \cdot F_3^2 + F_5^2 \cdot F_1^2 + F_3^2 \cdot F_5^2 \\ x = F_1^2 \cdot F_3^2 + F_5^2 \cdot F_1^2 + F_3^2 \cdot F_5^2 \\ y = F_1^2 \cdot F_3^2 \cdot F_5^2 \end{cases} \quad (16)$$

Table 1. Results according to equation (13).

f_e (MHz)	3004.392 ± 0.003	σ_{f_e}/f_e	10^{-6}
k_1 (%)	3.39 ± 0.04	σ_{k_1}/k_1	1.2×10^{-2}

Table 2. Measured structure parameters.

f_e (MHz)	3004.392 ± 0.003
f_a (MHz)	3005.44 ± 0.02
f_c (MHz)	2996.46 ± 0.03
k_1 (%)	3.39 ± 0.04
k_a (%)	-0.67 ± 0.15
k_c (%)	$-0.04 \pm \text{n.v.}$

These two equations give us *four invariants* which, combined *ad hoc*, may lead to the best estimate of the other unknowns f_a, f_c, k_a . All the parameter values are reported in table 2:

It has to be remembered that the results still depend on k_c , but because of its smallness, we do not expect a real influence. The unknown k_c is so small that the error is of the same order of magnitude as the variable.

In order to check the method, the measured frequency spectrum has been compared with the one obtained using the circuit simulation, as shown in figure 3.

In the worst case the difference between frequency values is roughly 400 kHz, 10^4 times smaller than the main frequency (roughly 3 GHz) and so well inside the acceptable resonance bandwidth.

The knowledge of the coupling constants allows series of measurements on a sample of cavities in order to validate the mechanical machining before the brazing process.

This can be done using a system of nine cavities, with three AC, four CC, and two end cavities, which are made symmetric. From the measurement of the nine mode frequencies and using the coupling constants previously measured, we can extract the value of the central AC cavity, by means of a symbolic code. A systematic procedure can be implemented with repeated measurements of a single cavity.

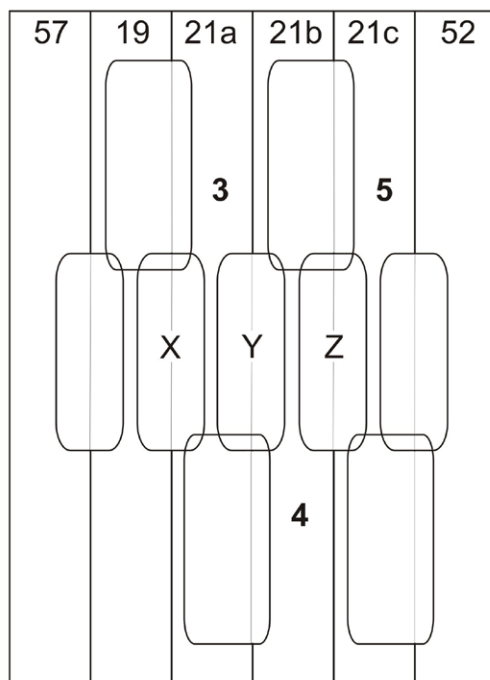


Figure 4. Nine-cavity scheme for central cavity measurements. Each tile is numbered on top.

Using the scheme of figure 4, we measure the cavity named Y; then one of tile (#3) is removed, shifting the adjacent tile (#4) to the left and replacing it with a new one. In this way we can measure a new accelerating cavity. Applying the procedure several times it is possible to measure all the ACs. The end cavities remain in their positions in order to properly close the chain.

This procedure allows control of the mechanical machining and eventual remachining of cavities which are not similar to the ideal one obtained from CAD simulation study on nine cavities.

In the case reported here, as an example, we got a reference AC frequency of 3007.550 MHz. Once measured, the cavities can be arranged in order to find an optimal configuration in which the central AC has a lower frequency with respect to the ideal cavity. The frequency can be always increased by means of tuners.

5. Conclusions

The coupling constants k_1 and k_a and single-cavity frequencies have been measured with very high accuracy. It is the first time that the constant k_a has been measured with such high accuracy. Even if the method has been applied to a subset

of cavities, the results are general and can be applied for the study of a real linac case.

The constant k_c can be evaluated with its error by means of a variational method, viable precisely because the error propagation has, as a starting point, precise data (those measured and those forced).

The method reported here can be used to characterize the coupling and the accelerating cavity tuners as a function of the progressive number (or fraction) of turns.

Furthermore, the possibility of controlling the structure parameters before the brazing of the linac is also important because all the following tests will be easier and more reliable: from the check on the working frequency to the accelerating electric field uniformity.

Finally, the accuracy of the frequency evaluation is 10^{-5} , more than one order of magnitude smaller than the value reported in equation (15).

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