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ECLIPSES INTERVALS FOR SATELLITES IN ELLIPTICAL ORBIT UNDER EFFECTS OF SOLAR RADIATION PRESSURE AND EARTH'S OBLATNESS

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ABSTRACT

The eclipse of a satellite in elliptical orbit is studied. The geocentric angle in the orbital plane between the perigee and conjunction point is determined. Eclipse times and duration of eclipse for elliptical orbits are obtained. The effects of the direct solar radiation pressure on the orbital elements are calculated taken into account the eclipses intervals. An application on the MOLNIYA 1-87 artificial satellite is done.

Key words: Perturbations Theory, Circular orbital motion, Earth's gravity, Radiation pressure effects, Shadow function.

INTRODUCTION

A number of authors studied implications for the movement of an artificial satellite due to the direct solar radiation pressure. Beginning from spearheading works, Musen (1960) deduced first order expressions for the rates of change in the orbital elements affected by the direct solar radiation pressure. Kozai (1961) and Brouwer (1962) utilized Lagrange's planetary equations to discover solutions of the first order with the complete integration between the times of egress from and ingress into the shadow. Anselmo et al. (1983) had dissected the effects of the solar radiation pressure as perturbation on the orbit of a high artificial satellite. Cook (2001) found that the most significant effect relating to solar radiation pressure is the changing cross-sectional area of the satellite projected to the Sun. Bar-Sever and Kuang (2005) introduced a set of solar pressure models for Global Positioning System (GPS) satellites based on in-orbit tracking data, and analyzed the performance of the models for satellites outside eclipse seasons. McMahon (2011) modeled the solar radiation pressure acceleration as a Fourier series which depends on the Sun's location in a body-fixed frame; a new set of Fourier coefficients are derived for every latitude of the Sun in this frame, and the series is expanded in terms of the longitude of the Sun. Hubaux et al. (2012) presented a creative method for spherical earth modeling umbra and penumbra cone crossings during the numerical integration of space debris orbit. Lücking et. al.(2012) further explored a passive strategy based on the joint effects of solar radiation pressure and the Earth's oblateness acting on a high area-to-mass-ratio object.

THE SHADOW INTERVALS

To obtain the shadow intervals it is more convenient to assume that

The Earth's shadow is circular cylinder with a diameter equal to the mean diameter of the Earth. Fig.(1)

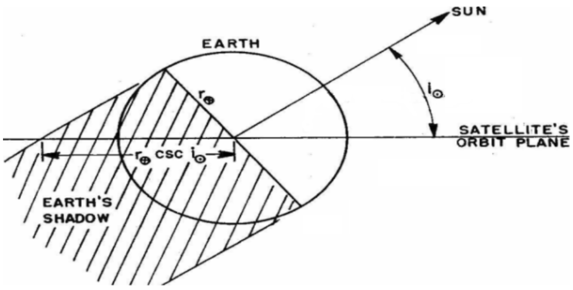

Fig. (1): The Earth's shadow with no penumbra (case of artificial satellites)

The shadow has no penumbra.

The atmospheric refractions are neglected

The synodic and sidereal periods are identical, owing to the small period of satellite.

Fig. (2) shows the ingress and egress of an artificial satellite to and from the shadow of Earth respectively.

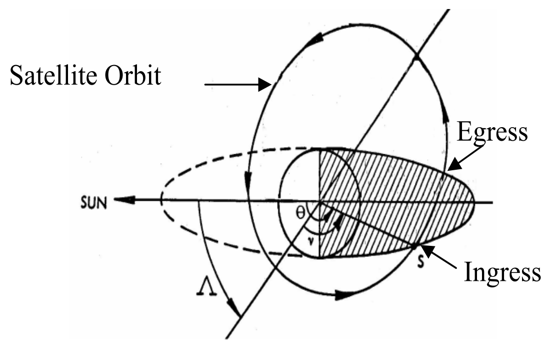

Fig. (2): Projection of Earth's shadow on the orbital plane of an elliptical orbit

Now, it is need to obtain the geocentric angle between the conjunction point and perigee $\Lambda\,\Lambda$ measured in the direction of the satellite's motion.

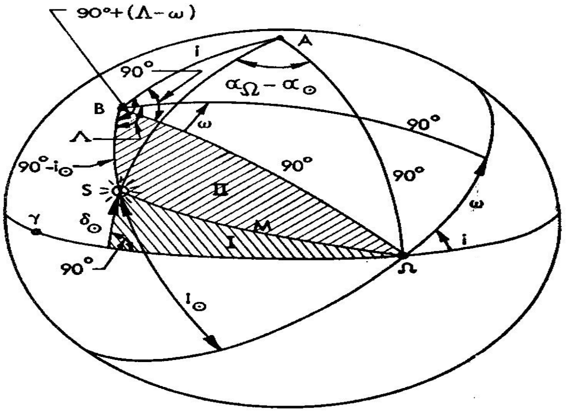

Fig. (3): Spherical triangles to determine $\Lambda\,\Lambda$

Using Fig. (3), it is clear from the spherical triangle (I)

$$\cos m = \cos\delta_{\odot}\cos(\alpha_{\Omega} - \alpha_{\odot}) \quad (1)$$

and from the spherical triangle (II)

$$\cos m = \cos i_{\odot}\cos(\Lambda - \omega) \quad (2)$$

From equation (1) and equation (2)

$$\cos(\Lambda - \omega) = \frac{\cos\delta_{\odot}\cos(\alpha_{\Omega} - \alpha_{\odot})}{\cos i_{\odot}} \quad (3)$$

Also, from the spherical triangle from S B A

$$\sin\delta_{\odot} = \sin i_{\odot}\cos i - \cos i_{\odot}\sin i\sin(\Lambda - \omega) \quad (4)$$

Then

$$\sin(\Lambda - \omega) = \frac{\sin i\cos i - \sin\delta_{\odot}}{\cos i\,\sin i} \quad (5)$$

The angle $(\Lambda - \omega)$ $(\Lambda - \omega)$ is obtained from Equations (4) and (5), then

$$\Lambda = (\Lambda - \omega) + \omega\Lambda = (\Lambda - \omega) + \omega \quad (6)$$

Equation (6) enables to obtain the angle between the conjunction point and perigee.

Eclipse times and duration of eclipse for elliptical orbits

Recall Fig. (2), the satellite enters or leaves the shadow when $r = r_{SH}$ and $\cos\theta$ can be obtained from

$$\cos\theta = \pm\frac{\sqrt{1-\left(\frac{r_{\oplus}}{r}\right)^2}}{\cos i} \quad (7)$$

For the value of

$$90^{\circ} < \theta < 180^{\circ} (8)\ 90^{\circ} < \theta < 270^{\circ}$$

Yields $\cos\theta$ to be negative, then the limits of the eclipse are obtained from Equation (7), then when $\theta = 180^{\circ}\theta = 180^{\circ}$, this yields

$$r = r_{\oplus}\csc i_{\odot}\ r = r_{\oplus}\cos e\, i_{\odot} \quad (9)$$

From Equation (8)

If $r < r_{\oplus}\csc i_{\odot}$, then an eclipse will occur

If $r > r_{\oplus}\csc i_{\odot}$,an eclipse will not occur

Since

$$r_{SH}^2 = \frac{r_{\oplus}^2}{1 - e_{SH}^2\cos^2\theta} \quad (10)$$

With e_{SH} , and $\theta = (\Lambda + \nu)$, then

$$r_{SH}^2 = \frac{r_{\oplus}^2}{1 - \cos^2 i_{\odot}\cos^2(\Lambda + \nu)} \quad (11)$$

Where ν is true anomaly

It is know that for elliptical orbits the trajectory equation is given as

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} \quad (12)$$

To find the true anomaly ν at which this condition is hold. Using Equation (12) with Equation (11), and solving for υ. After some little algebraic reduction this yields

$$k_1\cos^2(\Lambda + \nu) + k_2\cos^2\nu + k_3\cos\nu + k_4 = 0 \quad (13)$$

Where

$$k_1 = a^2(1 - e^2)\cos^2 i_{\odot} \quad (14.1)$$

$$k_2 = r_{\oplus}^2 e^2 \quad (14.2)$$

$$k_3 = 2\,e r_{\oplus}^2 \quad (14.3)$$

$$k_4 = r_{\oplus}^2 - a^2(1 - e^2)^2 \quad (14.4)$$

Equation (13) has four roots, and the only root at which the shadow occur that are hold the condition (8) which is $90^{\circ} < (\Lambda + \nu) < 270^{\circ}$. If no roots exist, hold the condition (8) no eclipse occurs.

To obtain the time passage of enter and leave eclipse, the following procedure is used.

The eccentric anomaly is calculated from

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2} \quad (15)$$

The mean anomaly is calculated by solving Kepler's Equation,

If the anomalies are expressed in degrees, this becomes

$$M = E - 57^{\circ}.29578\,(e\sin E) \quad (16)$$

This formula is used if the anomalies are expressed in degree

The times of the ingress and agrees are obtained from

$$t_i = T_p + \left(\frac{M_i}{360^\circ}\right) P \qquad (17)$$

$$t_e = T_p + \left(\frac{M_e}{360^\circ}\right) P \qquad (18)$$

Where, T_p is the time of perigee passage, and P is the period.

Equation (17) gives the time of the beginning the eclipse, Equation (18) gives the time of the ending of the eclipse, and the difference between them gives the duration of the eclipse.

Effects of solar radiation pressure and Earth's oblatness

Effects of Solar radiation pressure

The direct effect of the solar radiation on the satellite means the net acceleration resulting from the interaction (i.e. absorption, reflecting, or diffusion) of the sun light with each elementary surface of the spacecraft. Each photon carries an amount of momentum given by

$$M_{om} = \frac{E_g}{c} \qquad (19)$$

Where, M_{om} is the photon's momentum, E_g is the energy of the photon (proportional to the photon frequency and C is the velocity of light.

The momentum can be exchanged during interaction with a solid surface. So, the light behaves like a medium of material particles continuously emitted by the sun.

1`q 3`1A satellite whose surface has a reflection coefficient α, placed at a distance d from the sun and receiving the solar radiation at an angle of incidence ψ will experience an acceleration under the influence of solar radiation pressure, determined by

$$\bar{F} = -\frac{\beta_1}{d^2}\bar{R}_\odot \qquad (20)$$

$$\beta_1 = \frac{A}{m}\frac{\Phi_0}{c}(1+\alpha)a_\odot^2\cos^2\psi \qquad (21)$$

Where Φ_0 is the solar constant, $a_\odot$ $a_\odot$ is the mean distance Earth-Sun, and $\bar{R}_\odot$ is a unit vector in the direction Earth-Sun given in a geocentric equatorial frame by

$$\bar{R}_\odot = \cos\Lambda_\odot\,\bar{i} + \cos\varepsilon\sin\Lambda_\odot\,\bar{j} + \sin\varepsilon\sin\Lambda_\odot\,\bar{k} \qquad (22)$$

Where, ε is the obliquity of the ecliptic, $\Lambda_\odot$ is the true celestial longitude of the Sun. $\Lambda_\odot$ is expressed in terms of the orbital elements as $\Lambda_\odot = f_\odot + \omega_\odot$

$$\bar{F}_i = -\beta_1\frac{\bar{R}_\odot}{d^2} \qquad (23)$$

Where, assuming suitable averages of $\Lambda_\odot$, ψ and β_1 may considered constant.

Effects of Earth's oblatness

It is well known that the perturbing potential V depending on time and the position of the satellite, including the aspheristy of the earth is given by

$$V = -\frac{\mu}{r} + \frac{\mu}{r}\sum_{n=2}^{\infty} J_n\left(\frac{R}{r}\right)^n P_n(\cos\theta) \qquad (24)$$

where $P_n(\cos\theta)$ are the associated Legendre polynomial taken up to J_6 and

$\cos\theta = \frac{X_s}{r}$, r is the altitude of the satellite.

RESULT AND DISCUSSION

NUMERICAL EXAMPLE

An application is done to obtain the effect of direct radiation pressure taken into account the eclipses intervals on the artificial satellite MOLNIYA 1-87. The data of the MOLNIYA 1-87 is obtained from http://nssdc.gsfc.nasa.gov/nmc/spacecraftOrbit.do?id=1993-079A

From NORAD Two-Line Element Sets Current Data (MOLNIYA 1-87 epoch date 2005-08-31 02:01:13 UTC) is

1 22949U 93079A 05243.08418563 .00001291 00000-0 10000-3 0 7190

2 22949 64.2289 206.5639 6559220 260.5831 25.2033 2.00855470 85711

Using Lagrange's planetary equation in the form

$$\delta a = \frac{2e\sin f}{n(1-e)^{\frac{1}{2}}}Q + \frac{2(1+e\cos f)}{n(1-e)^{\frac{1}{2}}}T \qquad (25)$$

Where the variations in semi major axis due to the perturbations of the total incident radiations and thermal emission, only the radial Q and the transverse T components contribute to this variation

In an orthogonal reference frame with the origin in the center of the Earth, the x-axis pointing towards the perigee of the satellite orbit and the z-axis directed as the satellite orbital angular momentum, the radial, in-plane transverse, and out of plan unit vectors are written as

$$q = (\cos f, \sin f, 0) \qquad (26.1)$$

$$t = (-\sin f, \cos f, 0) \qquad (26.2)$$

$$w = (0,0,1) w = (0,0,1) \qquad (26.3)$$

Where a is the semi major axis, f is the true anomaly, e is the eccentricity, and n is the mean motion of the satellite orbit. Then, the components of the force can be written as:

$$S(f) = F(s).q S(f) = F(s).q \qquad (27.1)$$

$$T(f) = F(s).t T(f) = F(s).t \qquad (27.2)$$

Substitute from equation (26) into equation (24) yields the Lagrange planetary Equations in the Gaussian form,

$$\delta a = 2\, n\, a^3 (1-e^2)^{\frac{-1}{2}} F[e\, S(f) \sin f + T(\qquad (28.1)$$

$$\delta e = n\, a^2 (1-e^2)^{\frac{1}{2}} F\{ S(f) \sin f + T(f) \cos f \qquad (28.2)$$

$$\delta i = n\, a^2 (1-e^2)^{\frac{-1}{2}} FW \cos(f)\, \frac{r}{a} \cos u \qquad (28.3)$$

$$\delta\Omega \sin i = n\, a^2 (1-e^2)^{\frac{-1}{2}} FW\, \frac{r}{a} \sin u$$

$$\delta\, \Omega \sin i = na^2 (1-e^2)^{-\frac{1}{2}}\, FW\, \frac{?}{a} \sin u \qquad (28.4)$$

$$\delta\omega = -\delta\Omega \cos i + \frac{na^2(1-e^2)^{\frac{1}{2}}}{e} F\left[-S(f)\sin v + T(f)\left(1+\frac{r}{p}\right)\sin i\right]$$

$$\delta\omega = -\delta\Omega \cos i + \frac{na^2(1-e^2)^{\frac{1}{2}}}{e} F\left[-S(f)\cos v + T(f)\left(1+\frac{r}{p}\right)\sin i\right]$$

(28

$$\delta M = n - \frac{na^2 F\, S(f) r}{a} - (1-e^2)^{\frac{1}{2}} (\delta\omega + \delta\Omega \cos i) \qquad (28.6)$$

RESULTS

A cod is constructed using the Mathematica language to obtain the times of ingress and egress the shadow of the Earth respectively, the effects of the radiation pressure on the orbital elements of the satellites, and the oblatness of the Earth using Runge Kotta fourth order method

It is known that, the effect of shadow on the orbit of satellite is very important, so that, the results obtained illustrate that the satellite ingress the shadow at 148°.8739 and egress the shadow at 191°.415, at this interval the effect of the solar radiation pressure is vanishing, only the effect of the oblatness remain. The variations of the orbital elements are shown in figures (4) up to (9). It is clear that the variations about 10^{-6} which in agreement. Notice that, at the shadow interval the variation is approached to zero.

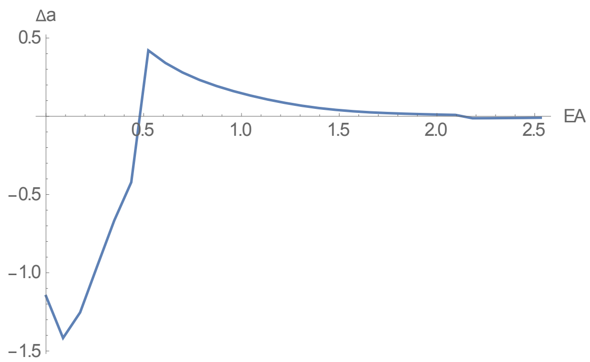

Fig. (4): Effects of R.P and oblatness on the semi major axis

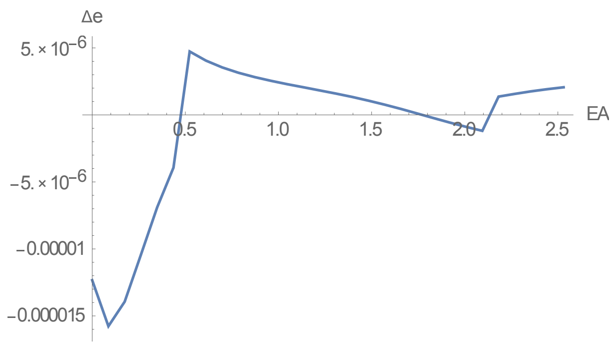

Fig. (5): Effects of R.P and oblatness on the eccentricity

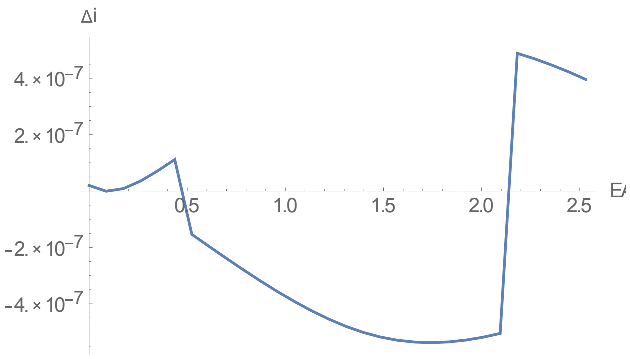

Fig. (6): Effects of R.P and oblatness on the inclination

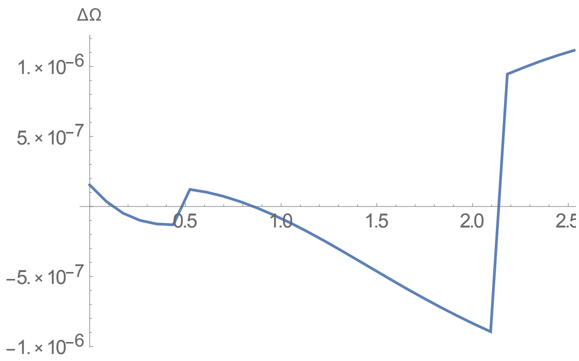

Fig (7): Effects of R.P and oblatness on the ascending node

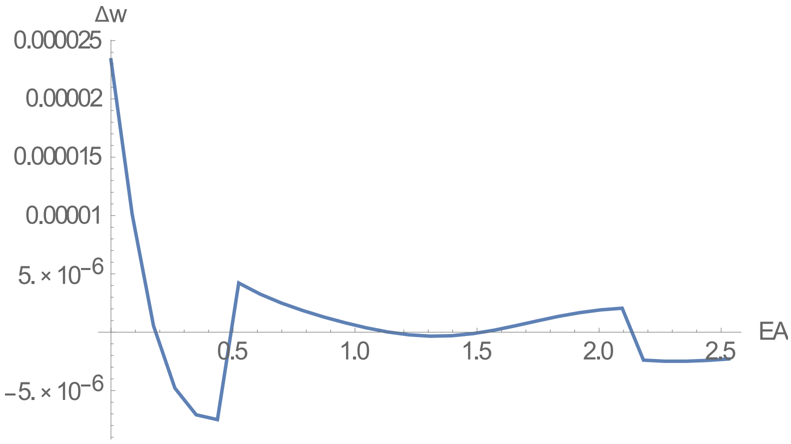

Fig (8): Effects of R.P and oblatness on the perigee

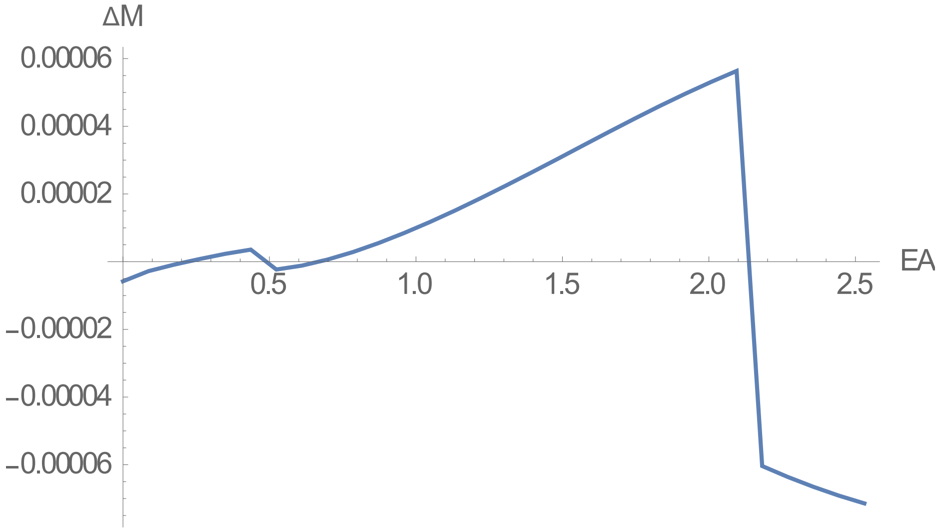

Fig (9): Effects of R.P and oblatness on the mean anomaly

CONCLUSION

This study illustrates the specification of the shadow interval during the motion of the satellites, also, when the satellite ingress and egress the shadow, this enables to take into account at the interval some instruments on the satellite are affected, which clear the importance of this study.

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