Some New Results of Residual and Past Entropy Measures

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Authors' contributions

This work was carried out in collaboration among all authors. Author AB designed the study, derived the mathematical expression and numerical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors ZI and En –BL managed the study and literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, two new generalized entropies have been introduced with their respective properties. The results of these entropies have been verified for the exponential and weighted exponential distributions. These two entropies produce the results in the form of simple entropy, generalized entropy, residual entropy, cumulative entropy and mixtures of all these entropies. Some characteristics of residual & past entropy have been derived and special cases have also been obtained. These cases indicate that new generalized entropies are more comprehensive and useful. The main advantage of this study is to derive different types of generalization of entropies using the different parameter values of α and β.

Keywords: Shannon entropy (Simple entropy); residual entropy; past entropy; exponential distribution.

1 Introduction

Entropy is widely used to measure the maximum disordering of a system. Entropy is the average of an information function. Shannon [1] was the first scientist who introduced the concept of entropy. As

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information function is the measure of amount of information then entropy is the average of the amount of information.

Renyi [2], Varma [3], Havrda and Charvat [4], Kapur [5], Arimoto [6], Sharma and Taneja [7], Sharma and Mittal [8], Awad et al. [9] and Tsallis [10] derived the different generalizations of the entropy.

Ebrahimi [11], who first introduced the residual entropy. Residual entropy is widely used in the life sciences and engineering. It is useful in the situation when uncertainty of any component of a system is measured by the information about its current age. Residual entropy is based on the survival function or the distribution function of the life distributions. Ebrahimi [11], Crescenzo and Longobardi [12], Rao et al. [13], Renyi (2004), Crescenze [14], Sunoj and Lino [15] give the different residual and past entropies which are expressed in equations (1 – 6) respectively. In the following equations $f(x), F(x)$ and $F(t)$ represents the probability density function, distribution function and distribution function at specific time 't' respectively.

$$H(F,t) = -\int_{t}^{\infty} \frac{f(x)}{1-F(t)} \ln\left(\frac{f(x)}{1-F(t)}\right) dx \tag{1}$$

$$\overline{H}(F,t) = -\int_{0}^{t} \frac{f(x)}{F(t)} \ln\left(\frac{f(x)}{F(t)}\right) dx \tag{2}$$

$$\varepsilon(X) = -\int_{0}^{\infty} \overline{F}(x) \ln \overline{F}(x) dx \tag{3}$$

$$H_{\alpha}(X) = \frac{1}{1-\alpha} \ln \int_{t}^{\infty} \left\{\frac{f(x)}{1-F(t)}\right\}^{\alpha} dx; \quad \alpha \neq 1 \tag{4}$$

$$\wp(F,t) = -\int_{0}^{t} \frac{F(x)}{F(t)} \ln\left(\frac{F(x)}{F(t)}\right) dx \tag{5}$$

$$H_{\beta}(X) = \frac{1}{1-\beta} \ln \int_{0}^{\infty} \left(1 - F(x)\right)^{\beta} dx; \quad \beta \neq 1 \tag{6}$$

In the literature different results of entropy, past entropy, residual entropy, cumulative entropy and generalized entropies are presented with their respective applications. Basit and Iqbal [16], Ebrahimi and Kirmani [11], Gupta and Nanda (2002), Abrham and Sankaran (2006), Nanda and Paul (2006) and Baig and Javid [17,18] gave the detailed overview on the residual entropy. Sunoj and Lino [15,19], Sunoj et al. [20], Sati and Gupta [21], Thapliyal and Taneja [22], Thapliyal et al. [23] produced their research on dynamic residual entropy, cumulative residual Tsallis entropy, Renyi entropy of order statistics and dynamic cumulative residual entropy. Ramdan [24], developed weighted entropy, weighted residual entropy and weighted pas entropy.

The comparison of entropies is also mentioned in the literature. Dey et al. [25], Mahdy and Eltelbany [26], Maszczyk and Duch [27] Majumdar and Sood [28], and Basit et al. [29] compared different entropies for different life distributions.

In the last two decades, entropy is widely use in different fields of sciences and social sciences. Kovalev [30] described the role of entropy in economics and identified its misuse. Purvis et al. [31] explained three different definitions of entropy in the field of economics. They produced the application of entropy to urban

system. Martos et al. (2018) studied stochastic process and derived some different entropies with their applications. Chen et al. [32] studied the sample entropy and permutation entropy with reference to the measuring complexity of time series. Lin and Oyapero [33] used the entropy and wavelet variance for analyzing the sequence of DNA. Sheraz et al. [34] and Dionisio et al. [35] studied the financial markets volatility using the different entropy measures.

In this paper, we are introducing two generalized entropies denoted by $A_{(\alpha,\beta)_1}(X)$ and $A_{(\alpha,\beta)_2}(t)$. The entropy $A_{(\alpha,\beta)_1}(X)$ produces different generalization of entropy as well as different cumulative entropies. Similarly, the entropy $A_{(\alpha,\beta)_2}(t)$ is producing different generalized residual entropies. The main reason of introducing these entropies is to include the simple entropy, generalized entropy, residual entropy and mixture of generalized and cumulative entropies. These entropies are more useful for the complex systems because these entropies are based on density function and survival function of the life distribution. These entropies easily provide the information of simple system as well as complex system. These entropies are more applicable for the systems where the hazard rate is in complex form. The remarks and theorem derived in section 2 and 3 described that new entropies are the generalization of some entropies developed previously.

In section 2; new generalized class of entropy has been introduced. The result of residual and past entropies for weighted exponential and exponential distribution are also described in section 2. The properties of new entropies and comparison are given in section 3.

2 New Class of Entropy Measures

In this section, we introduced two new classes of generalized entropies. These entropies give some particular cases of generalized entropies and residual & past entropies. If 'X' has absolutely continuous distribution function 'F' then the new generalized entropy is expressed in (7) and generalized residual entropy is expressed in (8).

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\beta)(1-\alpha)} \ln \int_0^\infty \{f(x)\}^\alpha \{1-F(x)\}^\beta dx \qquad , \alpha \& \beta \neq 1 \qquad (7)$$

$$A_{(\alpha,\beta)_2}(t) = \frac{1}{(1-\beta)(1-\alpha)} \ln \int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx \qquad , \alpha \& \beta \neq 1 \qquad (8)$$

Remark 2.1 Renyi [2], Rao et.al. [13], Shannon [1] and Sunoj and Linu [15] entropies are the particular cases of the $A_{(\alpha,\beta)_1}(X)$.

I.   Renyi [2] generalized entropy is the particular case of $A_{(\alpha,\beta)_1}(X)$ when β=0.

II.   Rao et.al. [13] entropy is a special case of $A_{(\alpha,\beta)_1}(X)$ when $\alpha=0, \beta \to 1$.

III. Shannon [1] entropy is a special case of $A_{(\alpha,\beta)_1}(X)$ when $\beta=0, \alpha \to 1$

IV.   Sunoj and Linu [15] entropy is the particular case of $A_{(\alpha,\beta)_1}(X)$ when $\alpha = 0$.

Remark 2.2 Renyi (2004) entropy is a particular case of $A_{(\alpha,\beta)_2}(X,t)$ when $\beta=-\alpha$.

$$(1+\alpha)A_{(\alpha,\beta)_2}(X,t)=\frac{1}{1-\alpha}\ln\int_t^\infty\left\{\frac{f(x)}{(1-F(t))}\right\}^\alpha dx$$

$$(1+\alpha)A_{(\alpha,\beta)_2}(X,t)=\mathrm{Re}\,nyi(2004) \tag{9}$$

Example 2.1 Let 'X' follows exponential distribution with pdf $f(X,\theta)=\theta e^{-\theta x}$ and distribution function $F(X,\theta)=1-e^{-\theta x}$. Then $A_{(\alpha,\beta)_1}(X)$ holds the following result.

$$A_{(\alpha,\beta)_1}(X)=\frac{\ln\left(\frac{\theta\alpha-1}{\alpha+\beta}\right)}{(1-\beta)(1-\alpha)}\;;\;\alpha\,\&\,\beta>0\,\&\,\neq 1$$

$$A_{(\alpha,\beta)_1}(X)=\frac{1}{(1-\beta)(1-\alpha)}\ln\left(\frac{\theta\alpha-1}{\alpha}\right)-\frac{1}{(1-\beta)(1-\alpha)}\ln\left(1+\frac{\beta}{\alpha}\right) \tag{10}$$

Example 2.2 Let 'Xʷ' follows weighted exponential distribution with pdf $g(X,\theta)$ and distribution function $G(X,\theta)$. Then $A_{(\alpha,\beta)_1}(X_w)$ holds the following result.

$$A_{1(\alpha,\beta)}(X)=\frac{1}{(1-\alpha)(1-\beta)}\ln\left\{\theta^{\alpha-1}\Gamma(\alpha+1)U(\alpha+1,\,\alpha+\beta+2,\,\alpha+\beta)\right\} \tag{11}$$

where

$$g(X;\theta)=\frac{W(X)f(X,\theta)}{E(W(X))}=\theta^2 xe^{-\theta x};W(X)=X_w=x\,;\,F(X;\theta)=1-\left(x+\frac{1}{\theta}\right)\theta e^{-\theta x}$$

$U(\alpha+1,\,\alpha+\beta+2,\,\alpha+\beta)$ is Tricomi confluent hypergeometric function and

$$U[\lambda,\delta,w]=\frac{1}{\Gamma\lambda}\int_0^\infty(1+s)^{\delta-\lambda-1}t^{\lambda-1}e^{-ws}ds$$

Example 2.3 Let 'X' follows exponential distribution with pdf $f(X,\theta)=\theta e^{-\theta x}$ and distribution function $F(X,\theta)=1-e^{-\theta x}$. Then $A_{(\alpha,\beta)_2}(X,t)$ holds the following results.

$$A_{2(\alpha,\beta)}(X) = \frac{1}{(1-\alpha)(1-\beta)} \left\{ \ln\left(\frac{\theta^{\alpha-1}}{\alpha}\right) - (\alpha+\beta)\theta t \right\} \tag{12}$$

$$(1-\alpha)(1-\beta)A_{2(\alpha,\beta)}(X,t) = (1-\alpha)H_\alpha(X) - (\alpha+\beta)r(t)t \tag{13}$$

where $r(t) = \dfrac{f(t,\theta)}{\overline{F}(t,\theta)}$ is the hazard function and $H_\alpha(X)$ is Renyi (1961) generalized entropy.

Example 2.4 Let 'X^w' follows weighted exponential distribution with pdf $g(X,\theta)$ and distribution function $G(X,\theta)$. Then $A_{(\alpha,\beta)_2}(X,t)$ holds the following result.

$$(1-\alpha)(1-\beta)A_{2(\alpha,\beta)}(X_w,t) = -\beta r(t_w) + (1-\alpha)H_\alpha(X_w)$$
$$+ \ln\left\{ \frac{Gamma[\alpha+1,t\alpha\theta]}{\Gamma(\alpha+1)} \right\} \tag{14}$$

where $r(t_w) = \dfrac{g(t,\theta)}{\overline{G}(t,\theta)}$ is the hazard function and $H_\alpha(X_w)$ is Renyi [2] generalized entropy of weighted exponential distribution respectively.

Theorem 2.1 Let 'X' is a positive continues random variable having pdf $f(X)$ and distribution function $F(X)$. Then $A_{(\alpha,\beta)_1}(X)$ is expressed as follows:

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\beta)}H_\alpha(X) + C \tag{14}$$

where C is constant and $C < 0$ for the distribution having hazard rate is constant and $\alpha \& \beta < 1$.

Proof:

Consider L.H.S.

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \int_0^\infty \{f(x)\}^\alpha \{1-F(x)\}^\beta dx$$

As

$$(1-z)^{-r} = \sum_{\delta=0}^\infty \frac{\Gamma(r+\delta)}{\Gamma r} \frac{z^\delta}{\delta!}; \ 0 \le Z \le 1$$

$$\Rightarrow \left(1-F(x)\right)^{\beta} = \sum_{\delta=0}^{\infty} \frac{\Gamma(-\beta+\delta)}{\Gamma(-\beta)} \frac{\left(F(x)\right)^{\delta}}{\delta!} ; \ 0 \le Z \le 1; 0 < \beta < 1$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \sum_{\delta=0}^{\infty} \frac{\Gamma(-\beta+\delta)}{\Gamma(\delta+1)\Gamma(-\beta)} \int_0^{\infty} \{f(x)\}^{\alpha} \left(F(x)\right)^{\delta} dx$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \left\{ \int_0^{\infty} \{f(x)\}^{\alpha} dx + \sum_{\delta=1}^{\infty} \frac{\Gamma(-\beta+\delta)}{\Gamma(\delta+1)\Gamma(-\beta)} \int_0^{\infty} \{f(x)\}^{\alpha} \left(F(x)\right)^{\delta} dx \right\}$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \left\{ \int_0^{\infty} \{f(x)\}^{\alpha} dx + \sum_{\delta=1}^{\infty} \frac{1}{\delta B(\delta,-\beta)} \int_0^{\infty} \{f(x)\}^{\alpha} \left(F(x)\right)^{\delta} dx \right\}$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \left\{ E\left\{ (f(x))^{\alpha-1} \right\} + \sum_{\delta=1}^{\infty} \frac{1}{\delta B(\delta,-\beta)} E\left\{ (f(x))^{\alpha-1} \left(F(x)\right)^{\delta} \right\} \right\}$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln \left[E\left\{ (f(x))^{\alpha-1} \right\} \left\{ 1 + \sum_{\delta=1}^{\infty} \frac{1}{\delta B(\delta,-\beta)} \frac{E\left\{ (f(x))^{\alpha-1} \left(F(x)\right)^{\delta} \right\}}{E\left\{ (f(x))^{\alpha-1} \right\}} \right\} \right]$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\alpha)(1-\beta)} \ln E\left\{ (f(x))^{\alpha-1} \right\} +$$

$$\frac{1}{(1-\alpha)(1-\beta)} \left[\ln \left\{ 1 + \sum_{\delta=1}^{\infty} \frac{1}{\delta B(\delta,-\beta)} \frac{E\left\{ (f(x))^{\alpha-1} \left(F(x)\right)^{\delta} \right\}}{E\left\{ (f(x))^{\alpha-1} \right\}} \right\} \right] \tag{15}$$

$$A_{(\alpha,\beta)_1}(X) = \frac{1}{(1-\beta)} H_{\alpha}(X) + C$$

From eq (10) and (15)

$$C = \frac{-1}{(1-\alpha)(1-\beta)} \left[\ln \left\{ 1 + \sum_{\delta=1}^{\infty} \frac{1}{\delta B(\delta,-\beta)} \frac{E\left\{ (f(x))^{\alpha-1} \left(F(x)\right)^{\delta} \right\}}{E\left\{ (f(x))^{\alpha-1} \right\}} \right\} \right] \tag{16}$$

Theorem 2.2 Let 'X' is a positive continues random variable having pdf $f(X)$ and distribution function $F(X)$. Then $A_{(\alpha,\beta)_2}(X,t)$ is expressed as follows:

$$(1-\alpha)(1-\beta)\frac{\partial A_{2(\alpha,\beta)}(X,t)}{\partial t} = \frac{e^{-(1-\alpha)(1-\beta)A_{2(\alpha,\beta)}(X,t)}}{\{S(t)\}^{-\beta}\{f(t)\}^{-\alpha}}\left\{-\beta r(t)f(t)^{-\alpha}\int_t^\infty \{f(x)\}^\alpha dx - 1\right\} \quad (17)$$

where

$$r(t) = \frac{f(t,\theta)}{\overline{F}(t,\theta)} , \ S(t) = 1 - F(t) \ \text{and} \ f'(t) = \frac{\partial f(t)}{\partial t}$$

Proof:

Consider eq (8)

$$A_{(\alpha,\beta)_2}(t) = \frac{1}{(1-\beta)(1-\alpha)}\ln\int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx \qquad ,\alpha \& \beta \neq 1$$

$$(1-\beta)(1-\alpha)A_{(\alpha,\beta)_2}(t) = \ln\int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx$$

Partially differentiate w.r.t. 't'

$$\frac{\partial(1-\beta)(1-\alpha)A_{(\alpha,\beta)_2}(X,t)}{\partial t} = \frac{1}{\int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx}\frac{\partial}{\partial t}\int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx$$

$$\frac{\partial(1-\beta)(1-\alpha)A_{(\alpha,\beta)_2}(X,t)}{\partial t} = \frac{1}{e^{(1-\alpha)(1-\beta)A_{(\alpha,\beta)_2}(X,t)}}\frac{\partial}{\partial t}\int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx$$

$$= \frac{1}{e^{(1-\alpha)(1-\beta)A_{(\alpha,\beta)_2}(X,t)}}\left[-\frac{\beta S(t)^\beta f(t)}{S(t)}\int_t^\infty \{f(x)\}^\alpha dx + S(t)^\beta \frac{\partial}{\partial t}\int_t^\infty \{f(x)\}^\alpha dx\right]$$

$$= \frac{S(t)^\beta}{e^{(1-\alpha)(1-\beta)A_{(\alpha,\beta)_2}(X,t)}}\left[-\beta r(t)\int_t^\infty \{f(x)\}^\alpha dx + \{-f(t)\}^\alpha\right]$$

$$= \frac{S(t)^\beta f(t)^\alpha}{e^{(1-\alpha)(1-\beta)A_{(\alpha,\beta)_2}(X,t)}} \left[-\frac{\beta r(t)}{f(t)^\alpha} \int_t^\infty \{f(x)\}^\alpha dx - 1 \right]$$

$$= \frac{S(t)^\beta f(t)^\alpha}{e^{(1-\alpha)(1-\beta)A_{(\alpha,\beta)_2}(X,t)}} \left[-\beta r(t) f(t)^{-\alpha} \int_t^\infty \{f(x)\}^\alpha dx - 1 \right] \tag{18}$$

Hence (18) is equal to the R.H.S of (17).

3 Properties

In this section we derived some linkages between the new entropies and Renyi [2] entropies.

Theorem 3.1 Let 'X' is a positive continues random variable having pdf $f(X)$ and distribution function $F(X)$. Then $A_{(\alpha,\beta)_1}(X)$ has the linear relationship with the Renyi [2].

$$(1-\beta) A_{(\alpha,\beta)_1}(X) = H_\alpha(X) + C \tag{19}$$

The result can be easily drawn from the theorem 2.1.

Theorem 3.2 Let 'X' is a positive continues random variable having pdf $f(X)$ and distribution function $F(X)$. Then $A_{(\alpha,\beta)_2}(X,t)$ has the linear relationship with the Renyi [2] i.e. $H_\alpha(X)$.

$$(1-\alpha)(1-\beta) A_{2(\alpha,\beta)}(X,t) = (1-\alpha) H_\alpha(X) + C(t) \tag{20}$$

where $C(t)$ is a function of truncated time 't'.

Proof:

Consider eq (8)

$$A_{(\alpha,\beta)_2}(t) = \frac{1}{(1-\beta)(1-\alpha)} \ln \int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx \qquad , \alpha \& \beta \neq 1$$

$$(1-\beta)(1-\alpha) A_{(\alpha,\beta)_2}(X,t) = \ln \int_t^\infty \{f(x)\}^\alpha \{1-F(t)\}^\beta dx$$

$$= \ln\left\{1 - F(t)\right\}^{\beta} + \ln \int_{t}^{\infty} \left\{f(x)\right\}^{\alpha} dx$$

$$= \ln\left\{1 - F(t)\right\}^{\beta} + \ln\left\{\int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx - \int_{0}^{t} \left\{f(x)\right\}^{\alpha} dx\right\}$$

$$= \ln\left\{1 - F(t)\right\}^{\beta} + \ln \int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx + \ln\left\{1 - \frac{\int_{0}^{t} \left\{f(x)\right\}^{\alpha} dx}{\int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx}\right\}$$

$$= \ln\left\{1 - F(t)\right\}^{\beta} + (1-\alpha) H_{\alpha}(X) + \ln\left\{1 - \frac{\int_{0}^{t} \left\{f(x)\right\}^{\alpha} dx}{\int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx}\right\}$$

$$(1-\beta)(1-\alpha) A_{(\alpha,\beta)_2}(X,t) = (1-\alpha) H_{\alpha}(X) + \ln\left\{\left\{1 - F(t)\right\}^{\beta}\left(1 - \frac{\int_{0}^{t} \left\{f(x)\right\}^{\alpha} dx}{\int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx}\right)\right\}$$

(21)

By comparing (20) and (21), we get the $C(t)$. i.e.

$$C(t) = \ln\left\{\left\{1 - F(t)\right\}^{\beta}\left(1 - \frac{\int_{0}^{t} \left\{f(x)\right\}^{\alpha} dx}{\int_{0}^{\infty} \left\{f(x)\right\}^{\alpha} dx}\right)\right\} \tag{22}$$

4 Relative Loss

For the comparison of the entropies, relative loss has been calculated using the exponential and weighted exponential (Size-biased moment exponential) distributions.

$$RL = \frac{H(X) - H(Y)}{H(X)}$$

where $H(Y)$ is the entropy of size-biased moment exponential distribution and $H(X)$ is the entropy of exponential distribution.

Table 1. Relative loss α – β order entropies

β	θ=0.6, α = 1.2				θ=0.9, α = 1.2				θ=1.2, α = 1.2			
	V 1966	**K 1967**	**ST1975**	**New A1**	**V 1966**	**K 1967**	**ST1975**	**New A1**	**V 1966**	**K 1967**	**ST1975**	**New A1**
0.20	-0.202	0.013	-0.309	-0.297	-0.252	0.015	-0.334	-0.364	-0.306	0.016	-0.362	-0.434
0.40	-0.271	-0.104	-0.392	-0.240	-0.351	-0.124	-0.454	-0.280	-0.443	-0.144	-0.528	-0.317
0.60	-0.330	-0.222	-0.405	-0.209	-0.440	-0.280	-0.505	-0.237	-0.575	-0.344	-0.629	-0.262
0.80	--	-0.330	-0.385	-0.189	--	-0.440	-0.517	-0.211	--	-0.575	-0.692	-0.229
0.99	-0.426	-0.424	-0.351	-0.176	-0.595	-0.591	-0.509	-0.194	-0.829	-0.822	-0.729	-0.208
1.20	-0.470	-0.516	--	-0.165	-0.671	-0.755	--	-0.180	-0.965	-1.125	--	-0.192
1.40	-0.508	-0.596	-0.261	-0.157	-0.740	-0.908	-0.458	-0.170	-1.094	-1.446	-0.764	-0.181
1.60	-0.543	-0.667	-0.218	-0.151	-0.805	-1.057	-0.426	-0.163	-1.225	-1.804	-0.767	-0.172
1.80	-0.576	-0.732	-0.178	-0.146	-0.868	-1.201	-0.393	-0.157	-1.357	-2.204	-0.766	-0.165
2.00	-0.606	-0.790	-0.141	-0.142	-0.928	-1.341	-0.359	-0.152	-1.491	-2.652	-0.760	-0.159

Table 2. Relative loss of residual entropies

T	θ=0.6, α = β=1.2			θ=0.9, α = β=1.2			θ=1.2, α = β=1.2		
	E1996	LC2002	New A2	E1996	LC2002	New A2	E1996	LC2002	New A2
0.20	-0.373	-0.419	0.267	-0.499	-0.574	0.441	-0.659	-0.783	0.557
0.40	-0.357	-0.401	0.467	-0.464	-0.534	0.590	-0.596	-0.707	0.645
0.60	-0.339	-0.382	0.548	-0.430	-0.494	0.624	-0.539	-0.638	0.645
0.80	-0.323	-0.363	0.583	-0.399	-0.458	0.626	-0.490	-0.579	0.625
0.99	-0.307	-0.345	0.597	-0.373	-0.427	0.616	-0.451	-0.530	0.601
1.20	-0.292	-0.327	0.600	-0.347	-0.396	0.600	-0.413	-0.485	0.575
1.40	-0.278	-0.311	0.598	-0.325	-0.371	0.583	-0.383	-0.448	0.551
1.60	-0.265	-0.297	0.591	-0.306	-0.348	0.566	-0.357	-0.416	0.528
1.80	-0.254	-0.283	0.583	-0.289	-0.328	0.549	-0.334	-0.388	0.507
2.00	-0.243	-0.271	0.574	-0.273	-0.310	0.532	-0.313	-0.364	0.488

5 Conclusion

In this study, we derived two new classes of generalized entropy. These entropies produce some new results which are extensions of some previous entropies. Shannon [1], Renyi entropy ([2], 2004), Rao entropy [13] and Sunoj and Linu entropy (2010)[15] are the special cases of new generalized entropies. This study also describes the linear relationship between the simple entropy and weighted entropy. The main advantage of this study is to derive different types of generalization of entropies using the different values of parameters, α and β. These two entropies generalize the simple entropy, generalized entropy, cumulative entropy, generalized residual entropy and combinations of these different types of entropies. We expect to further study their applications in future projects.

Competing Interests

Authors have declared that no competing interests exist.

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