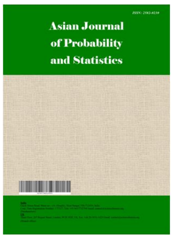

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Inverse Dominating Set of an Interval-valued Fuzzy Graphs

Ahmed N. Shain[1*] and Mahiuob M. Q. Shubatah[2*]

[1]*Department of Mathematic, Faculty of Education, Arts and Science, Sheba Region University, Marib, Yemen.*

[2]*Department of Mathematics, Faculty of Science and Education, AL-Baydaa University, AL-Baydaa, Yemen.*

Authors' contributions

This work was carried out in collaboration between both authors. Author ANS designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author MMQS managed the analyses of the study the literature searches. Both authors read and approved the final manuscript.

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Abstract

Inverse domination is very much useful in network theory, Radio Stations, Electrical stations and several fields of mathematics. In This article, inverse domination in an interval-valued fuzzy graphs is defined and studied. Some bounds on inverse domination number $\gamma^{`}(G)$. are provided for several interval-valued fuzzy graphs, such as complete, complete bipartite, star,.. etc. Furthermore, the relationship of $\gamma^{`}(G)$. with some others known parameters in interval-valued fuzzy graphs investigated with some suitable examples.

Keywords: Fuzzy graph, Interval-valued fuzzy graph, Inverse dominating set and inverse domination number.

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**Corresponding author: E-mail: ahmedshain942@gmail.com, mahioub70@yahoo.com;*

1 Introduction

Zadeh [1] introduced the idea of interval-valued fuzzy sets as an extension of fuzzy sets [2] which gives a more precise tool to model uncertainty in real life situations. Interval-valued fuzzy sets have been widely used in many areas of science and engineering. Rosenfeld introduced another detailed definition for each fuzzy vertex, fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness etc [3]. The concept of dominating in fuzzy graphs was investigated by A. Somasundaram and S. Somasundaram [4] and A. Somasundaram presented the concepts of independent domination, total domination, connected domination of fuzzy graphs [5]. The concept of domination in interval-valued fuzzy graphs introduced and investigated by Pradip Debnath [6]. The concept of inverse domination in fuzzy graphs introduced and investigated by S. Ghobadi, N.D. Soner and Q.M. Mahioub [7]. Because of the wide application of inverse domination in the real live, such as backup stations for radio broadcasting, reserve stations for electricity networks, computer networks and the internets network. The rapid growth of research in this area is due to the following: 1. Due to his importance in the practical side of electricity and internet networks...etc. 2. The wide variety of domination parameters that can be defined. 3. Due to the wide varity of applicaton of domination in fuzzy graphs. 4. Due to the work of domination on fuzzy graph and interval-valued fuzzy graph. In this paper we introduce and investigate the concept of inverse domination in interval-valued fuzzy graphs and we will obtain many result related to this concept.

2 Preliminaries

We reviewin this section some basic definitions related to interval-valued fuzzy graphs and inverse domination in fuzzy graph.

An interval-valued fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (A, B)$, where $A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set on V, $B = [\rho_B^-, \rho_B^+]$ is an interval-valued fuzzy relation on V, such that $\mu_A^-(x) \leq \mu_A^+(x); \forall x \in V$ and $\rho_B^-(x,y) \leq min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\rho_B^+(x,y) \leq min\{\mu_A^+(x), \mu_A^+(y)\}; \forall (x,y) \in E$. In an interval-valued fuzzy graph G, when $\rho^-(u,v) = \rho^+(u,v) = 0$ for some u and v, then there is no edge between u and v. Otherwise there exists an edge between u and v. Let $G = (A, B)$ be an interval-valued fuzzy graph. Then the cardinality of interval-valued fuzzy graph G is defined as

$$|G| = \sum_{u \in V} \frac{1 + \mu^+(u) - \mu^-(v)}{2} + \sum_{(u,v) \in E} \frac{1 + \rho^+(u,v) - \rho^-(u,v)}{2}.$$

The vertex cardinality of an interval-valued fuzzy graph G is defined by

$$|V| = \sum_{u \in V} \frac{1 + \mu^+(u) - \mu^-(u)}{2}.$$

For all $u \in V$ is called the order of an interval-valued fuzzy graph and is denoted by $p(G)$. The edge cardinality of an interval-valued fuzzy graph G is defined by

$$|E| = \sum_{(u,v) \in E} \frac{1 + \rho^+(u,v) - \rho^-(u,v)}{2}.$$

For all $(u,v) \in E$ is called the size of an interval-valued fuzzy graph and is denoted by $q(G)$. An edge $e = (x,y)$ of an interval-valued fuzzy graph is called effective edge if $\rho^+(x,y) = min\{\mu^+(x), \mu^+(y)\}$ and $\rho^-(x,y) = min\{\mu^-(x), \mu^-(y)\}$. The degree of a vertex can be generalized in different ways for an interval-valued fuzzy graph $G = (A, B)$. The effective degree of a vertex v in an interval-valued fuzzy graph, $G = (A, B)$ is defined to be summation of the weights of the effective edges incident at v and it is denoted by $d_E(v)$. The minimum effictive edges degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$. A vertex u of an interval-valued fuzzy graph G is said to be an isolated vertex if $\rho^-(uv) < min\{\mu^-(u), \mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u), \mu^+(v)\}$ for all $v \in V - \{u\}$ such that there is an edge between u and v, i.e., $N(u) = \phi$. A set S of vertices of an interval-valued fuzzy graph G is said to be independent if $\rho^-(uv) < min\{\mu^-(u), \mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u), \mu^+(v)\}$ for all $u, v \in S$. An interval-valued fuzzy graph, $G = (A, B)$ is said to be Complete interval-valued fuzzy graph if $\rho^-(v_i, v_j) = min\{\mu^-(v_i), \mu^-(v_j)\}$, $\rho^+(u, v) = min\{\mu^+(u), \mu^+(v)\}$, for all $u, v \in V$ and denoted by K_p. The complement of an interval-valued fuzzy graph, $G = (A, B)$ is an interval-valued fuzzy graph, $\overline{G} = (\overline{A}, \overline{B})$ where $\overline{\rho^-}(u, v) = min\{\mu^-(u), \mu^-(v)\} - \rho^-(u, v)$ and $\overline{\rho^+}(u, v) = min\{\mu^+(u), \mu^+(v)\} - \rho^+(u, v)$ for all $u, v \in G$. An interval-valued fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\rho^-(xy) = 0$ and $\rho^+(xy) = 0$ if $x, y \in V_1$ or $x, y \in V_2$. Further if $\rho^-(xy) = min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\rho^+(xy) = min\{\mu_A^+(x), \mu_A^+(y)\}$ for all $x \in V_1$ and $y \in V_2$. Then G is called a complete bipartite fuzzy graph is denoted by $K_{\mu_A^-, \mu_A^+}$, where μ_A^- and μ_A^+ are restrictions of μ_A^- and μ_A^+ on V_1 and V_2 respectively. An edge $e = xy$ of an interval-valued fuzzy graph G is called an effective edge if $\rho^-(xy) = min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\rho^+(xy) = min\{\mu_A^+(x), \mu_A^+(y)\}$. In this case, the vertex x is called a neighbor of y and conversely. $N(x) = \{y \in V : y \text{ is a neighbor of} x\}$ is called the neighborhood of x. Let $G = (\mu, \rho)$ be a fuzzy graph and let S be minimal dominating set of G, if $V - D$ contains a dominating set $D^{`}$ of a fuzzy graph $G = (\mu, \rho)$, then $D^{`}$ is called an inverse dominating set of $G = (\mu, \rho)$ with respect to D. The minimum fuzzy cardinality taken over all inverse dominating sets of G is called the inverse dominating number of a fuzzy graph $G = (\mu, \rho)$ is denoted by $\gamma^{`}(G)$. A subset $(D \subseteq V)$ of $V(G)$ is called dominating set in interval-valued fuzzy graph G, if for every $v \in V - D$ there exists $u \in D$ Such that (u, v) is strong edge. A dominating set D of G is called minimal dominating set of G if $D - \{u\}$ is not dominating set for every $u \in D$. A minimal dominating set D, with $|D| = \gamma(G)$ is denoted by $\gamma - set$.

3 Inverse Dominating Set of an Interval-valued Fuzzy Graphs

Definition 3.1. Let $G = (A, B)$ be an interval-valued fuzzy graph and let S be $\gamma - set$ of G, if $V - S$ has a dominating set $S^{`}$. Then $S^{`}$ is called an inverse dominating set of G with respect to S.

Definition 3.2. An inverse dominating set $S^{`}$ of interval-valued fuzzy graph $G$, is called minimal inverse dominating set, if $S^{`} - \{u\}$ is not inverse dominating set, for every $u \in S^{`}$.

Definition 3.3. The minimum fuzzy cardinality taken over all minimal inverse dominating sets of an interval-valued fuzzy graph G, is called The inverse domination number of an interval-valued fuzzy graph G and denoted by $\gamma^{`}(G)$ or simply $\gamma^{`}$. The maximum fuzzy cardinality among all minimal inverse dominating set of an interval-valued fuzzy graph G is called the upper inverse domination number of G and is denoted by $\Gamma^{`}(G)$.

Definition 3.4. An inverse dominating set of an interval-valued fuzzy graph G, with $|S^{`}| = \gamma^{`}(G)$, is denoted by $\gamma^{`} - set$.

Example 3.1. *For a Strong interval-valued fuzzy graph G, Shown in Fig. 1.*

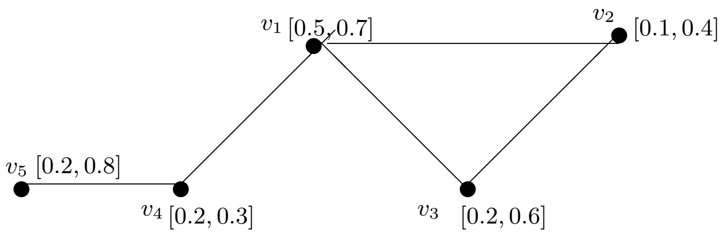

Fig. 1.

We have, $S = \{v_1, v_4\}$ *is dominating set, then* $V - S = \{v_2, v_3, v_5\}$*. Therefore,* $S_1' = \{v_2, v_5\}$ *and* $S_2' = \{v_3, v_5\}$ *are minimal inverse dominating sets. Hence* $\gamma'(G) = min\{|S_1'|, |S_2'|\} = 1.45$.

Remark 3.1.

(i) For any Complete interval-valued fuzzy graph K_p, $\gamma'(K_p) \leq \gamma'(K_{p-v})$. Where $v \in K_p$.

(ii) For any interval-valued fuzzy graph G with at least one inverse dominating set, $\gamma(G) \leq \gamma'(G)$.

Theorem 3.1. *For any interval-valued fuzzy graph* G,

$$\gamma'(G) + \gamma(G) \leq p.$$

Proof. Let G be an interval-valued fuzzy graph G, S is a $\gamma - set$ of G and S' be $\gamma'(G) - set$ with respect to S of G. $S' \subseteq V - S$. Then

$$|S'| \subseteq |V - S|.$$

So

$$\gamma'(G) \leq p - \gamma(G).$$

Hence

$$\gamma'(G) + \gamma(G) \leq p.$$

□

Theorem 3.2. *For any interval-valued fuzzy graph* G.

$$\gamma'(G) + \gamma'(\overline{G}) \leq 2p.$$

Proof. Since $\gamma'(G) \leq p$ and $\gamma'(\overline{G}) \leq p$. Then

$$\gamma'(G) + \gamma'(\overline{G}) \leq 2p.$$

□

Theorem 3.3. *For any interval-valued fuzzy graph* G*, with at least one isolated vertex,* $\gamma'(G) = 0$.

Proof. Let $G = (A, B)$ be any interval-valued fuzzy graph, x be an isolated vertex in G and let S be $\gamma - set$ of G. Therefore, $x \in S$.

Now, Suppose that G has an inverse dominating set say S' and $S' \neq \phi$. Then every vertex is not isolated.

Since $S' \subseteq V - S$. $y \in S'$. Then exists a vertex $x \in S$, such that x adjacent to y which is a contrdication. □

Theorem 3.4. *An inverse dominating set* S' *of an interval-valued fuzzy graph* G*, is minimal inverse dominating set if and only if one of the following condition holds:*

(1) $N(v) \cap S^{\backprime} = \phi$;

(2) There is a vertex $u \in V - S^{\backprime}$, *such that* $N(u) \cap S^{\backprime} = \{v\}$.

Proof. Let G be an interval-valued fuzzy graph with $S^{\backprime}$ be $\gamma^{\backprime}(G) - set$ of G and let $v \in S^{\backprime}$. Then $S^{\backprime} - \{v\}$ is not dominating set of G, so is not inverse dominating set of G and there exists a vertex $u \in V - \{S^{\backprime} - \{v\}\}$, such that u is not dominated by any vertex of $S^{\backprime} - \{v\}$. If $u = v$. Then $N(v) \cap S^{\backprime} = \phi$, if $u \neq v$. Then $N(u) \cap S^{\backprime} = \{v\}$.

Conversely: Suppose that $S^{\backprime}$ is an inverse dominating set of G and for each a vertex $v \in S^{\backprime}$ one of two conditions holds. Suppose that $S^{\backprime}$ is not minimal inverse dominating set. Then there exsits a vertex $v \in S^{\backprime}$; $S^{\backprime} - \{v\}$ is an inverse dominating set. Hence, v is adjacent to at least one vertex in $S^{\backprime} - \{v\}$. Then the condition one dose not hold. If $S^{\backprime} - \{v\}$ is an inverse dominating set. Then every vertex in $V - S^{\backprime}$ is adjacent to at least one vertex in $S^{\backprime} - \{v\}$. Then the condition two dose not hold, which a contradiction. So $S^{\backprime}$ is minimal inverse dominating set. □

Remark 3.2. For any interval-valued fuzzy graph G,

$$\gamma^{\backprime}(G) \leq \frac{p}{2}.$$

Example 3.2. *For the strong interval-valued fuzzy graph G shown in Fig. 2.*

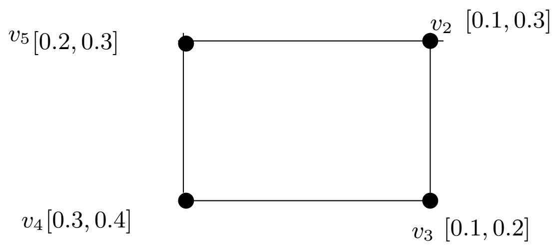

Fig. 2

In Fig. 2, we have $\gamma^{\backprime}(G) \leq \frac{p}{2}$.

Theorem 3.5. *Every covering set of an interval-valued fuzzy graph G is a dominating set.*

Proof. Let S be cover set of G. Then $\forall\ (x, y) \in E(G)$ ether $x \in S$ or $y \in S$. Therefor, $\forall\ u \notin S$ there exists $v \in S$ such that v dominates u. Then S is dominating set.

□

Theorem 3.6. *For any interval-valued fuzzy graph* G, *without isolated vertex,*

$$\gamma(G) \leq \gamma^{\backprime}(G).$$

Proof. Suppose that S is a vertex cover set of an interval-valued fuzzy graph G. Then $V - S$ is an independent set of G. By above Theorem, every covering set is a dominating set of G, thus S is a dominating set of G. Since G has no isolated vertex, then $V - S$ is dominating set of G. Then $V - S$ is an inverse dominating set of G. with respect to S. Hence,

$$\gamma(G) \leq |V - S| = \gamma^{\backprime}(G).$$

□

Theorem 3.7. *For any interval-valued fuzzy graph G, without isolated vertices,*

$$\gamma^{\backprime}(G) \leq \beta_0(G).$$

Proof. Let G be an interval-valued fuzzy graph and S be a maximal independent set. Then S is dominating set of G and $V-S$ is a vertex covering set of G. So $V-S$ is dominating set of G. Thus $V-S$ is an inverse dominating set in $IVFG$. Hence,

$$\gamma^{\backprime}(G) \leq |V-S| = p - \alpha_0(G) = \beta_0(G).$$

□

Corollary 3.1. *For any interval-valued fuzzy graph G, without isolated vertices,*

$$\gamma^{\backprime}(G) \leq p - \alpha_0(G).$$

Theorem 3.8. *For any interval-valued fuzzy graph G,*

$$\gamma^{\backprime}(G) \leq p - \Delta_N.$$

Proof. Let G be an interval-valued fuzzy graph and v be a vertex in G, with $d_N(v) = |N(v)| = \Delta_N$. Then $V - N(v)$ is a $\gamma - set$ and $S^{\backprime} \subseteq V - N(v)$. Hence,

$$|S^{\backprime}| \leq |V - N(v)| = p - |N(v)|.$$

Hence,

$$\gamma^{\backprime}(G) \leq p - \Delta_N.$$

□

Corollary 3.2. *For any interval-valued fuzzy graph G,*

$$\gamma^{\backprime}(G) \leq p - \Delta_E \quad \text{and } \gamma^{\backprime}(G) \leq p - \delta_N.$$

Proof. Since $\Delta_E \leq \Delta_N$ for any fuzzy graph and by above Theorem. Then $\gamma^{\backprime}(G) \leq p - \Delta_E$. Similarly, since $p - \Delta_N \leq p - \delta_N$. Hence $\gamma^{\backprime}(G) \leq p - \delta_N$. □

Corollary 3.3. *For any interval-valued fuzzy graph G,*

$$\gamma^{\backprime}(G) \leq p - \Delta_N^{+} \text{ and } \gamma^{\backprime}(G) \leq p - \Delta_N^{-}.$$

Proof. Since $\gamma^{\backprime}(G) \leq p - \Delta_N$ and $\Delta_N \geq \Delta_N^{+}$. Then $p - \Delta_N \leq p - \Delta_N^{+}$. Hence,

$$\gamma^{\backprime}(G) \leq p - \Delta_N^{+} \leq p - \Delta_N^{-}.$$

□

Remark 3.3. For any interval-valued fuzzy graph G,

$$\Delta_N^{+} + \Delta_N^{-} \leq p - 1.$$

Theorem 3.9. *For any interval-valued fuzzy graph G,*

$$\gamma^{\backprime}(G) \leq \frac{p+1}{2}.$$

Proof. since $\gamma^{`}(G) \leq p - \Delta_N^+$ and $\gamma^{`}(G) \leq p - \Delta_N^-$. Then

$$2\gamma^{`}(G) \leq 2p - \Delta_N^+ + \Delta_N^-.$$

$2\gamma^{`}(G) \leq 2p - (p-1) = p+1$. Hence,

$$\gamma^{`}(G) \leq \frac{p+1}{2}.$$

□

Theorem 3.10. *Let $G = (A, B)$ be a complete Interval-valued fuzzy graph. Then*

$$\gamma^{`}(G) = min\{|v|; v \in V - S\}.$$

Proof. Let S be dominating sets of an interval-valued fuzzy graph G. Then S contains only one vertex with minimum cardinality $\gamma(G) = min\{|v|; v \in V\}$.

Therefore, $V - S$ contains a dominating set $S^{`}$ such that $S^{`}$ contains only one vertex. Thus

$$\gamma^{`}(G) = min\{|v|;\ v \in V - S\}.$$

□

Theorem 3.11. *Let $G = (A, B)$ be a complete bipartite Interval-valued fuzzy graph. Then*

$$\gamma^{`}(G) = min\{|v|; v \in V_1 - S\} + min\{|u|; u \in V_2 - S\}.$$

Proof. Let S be dominating set of G. Then S contains only one vertex with minimum cardinality $\gamma(G) = min\{|v|; v \in V_1\} + min\{|u|; u \in V_2\}$. Therefore, $V - S$ contains a dominating set $S^{`}$ such that $S^{`}$ is contains only one vertecx. Thus

$$\gamma^{`}(G) = min\{|v|; v \in V_1 - S\} + min\{|u|; u \in V_2 - S\}.$$

□

Theorem 3.12. *For any interval-valued fuzzy graph G,*

$$\gamma^{`}(G) \leq \Gamma(G).$$

Furthemor equality holds if G is a path.

Proof. Let P be a path in interval-valued fuzzy graph G and S be a $\gamma^{`} - set$, Since every path contains only two dominating sets say S and $V - S$. Thus

$$S^{`} = V - S.$$

Hence,

$$\gamma^{`}(P) = |V - S| = \Gamma^{`}(P).$$

□

Theorem 3.13. *For any interval-valued fuzzy graph G, with at least one inverse dominating set. Then*

$$\gamma(G) \leq \frac{p + \gamma^{`}(G)}{3}.$$

Proof. Let $G = (A, B)$ be an interval-valued fuzzy graph , with $\gamma^{\backslash}(G) - set$, since $\gamma \leq \gamma^{\backslash}(G)$ and $\gamma \leq \frac{p}{2}$. Hence

$$\gamma(G) \leq \frac{p + \gamma^{\backslash}(G)}{3}.$$

□

Corollary 3.4. *For any interval-valued fuzzy graph G, with at least one inverse dominating set. Then*

$$\gamma^{\backslash}(G) \geq 3\gamma(G) - p.$$

Theorem 3.14. *Let $G = (A, B)$ be any interval-valued fuzzy graph and $H = (A^{\backslash}, B^{\backslash})$ is a partial interval-valued fuzzy subgraph of G. Then*

$$\gamma^{\backslash}(G) \leq \gamma^{\backslash}(H).$$

Proof. Let $G = (A, B)$ be an $IVFG$, $H = (A^{\backslash}, B^{\backslash})$ is a partial interval-valued fuzzy subgraph of G. Let S be $\gamma - set$ of G and $S^{\backslash}$ is $\gamma^{\backslash} - set$ of G. Then

$$\gamma(H) \leq \gamma(G).$$

Then

$$p - \gamma(G) \leq p - \gamma(H).$$

Hence

$$\gamma^{\backslash}(G) \leq \gamma^{\backslash}(H).$$

□

4 Conclusions

The concept of inverse domination number which denoted by $\gamma^{\backslash}(G)$ of an interval-valued fuzzy graph introduced and studied. Uper and lower boundeds of $\gamma^{\backslash}(G)$ in interval-valued fuzzy graph G are obtained. The relationship of $\gamma^{\backslash}(G)$ and some other known parameters in interval-valued fuzzy graph G studied. Some suitable examples are given.

Competing Interests

Authors have declared that no competing interests exist.

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