On the Improvement of Multivariate Ratio Method of Estimation in Sample Surveys by Calibration Weightings

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Original Research Article

Abstract

The most challenging limitation of the ratio estimation is that of deriving variance estimator that admits more than two auxiliary variables. This paper introduces a new calibration weights that prompt the formulation of a multivariate ratio estimator by the calibration tuning parameter subject to a pooled-calibration constraint. Analytical framework for deriving variance estimator that admits as many auxiliary variables as desired is developed. The efficiency gains of the proposed estimator vis-a-vis the Generalized Regression (*GREG*) Estimator are studied through simulation. Simulation results proved the dominance of the new proposals over existing ones.

1 Introduction

[1] first introduced the ratio estimation in survey sampling. It is well known that the ratio [and product] estimators have the limitation of having efficiency not exceeding that of the linear regression estimator.

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Consequently, since the discovery of ratio method of estimation; many authors have come up with various degrees of modifications of the ratio estimator for better performances. These authors include [2-13] among others.

In the progression for improved ratio estimators, [14] advocated the use of multiple auxiliary variables and proposed multivariate ratio estimator in simple random sampling. Following [14] estimator several other estimators using multiple auxiliary variables have been proposed by Researchers in Survey Theory. [15] has extended [14] estimator to the case where auxiliary variables are negatively correlated with the variable under study. Other authors [16-18] have equally proposed multivariate ratio estimators of various forms in survey sampling. The main objective of presenting these estimators was to reduce the bias and mean square errors.

[19] observed that most of these alternative ratio estimators depend on some optimality conditions that are hardly satisfy in practice and advocated the use of calibration estimation to address these problems. [20] first presented calibration estimators in survey sampling and calibration estimation has been studied by many Researchers in Survey Theory. A few key references include [21-29].

Several, related literature reviewed showed that tremendous work have been done on calibration estimation under the univariate ratio and univariate regression methods of estimation and much more on the multivariate regression estimation [see 30-32], but the theory of calibration estimator for multivariate ratio estimation is not well known.

Consequently, this paper is an attempt at developing the theory of calibration estimator for multivariate ratio method of estimation in simple random sampling without replacement (SRSWOR) and stratified random sampling designs by introducing new calibration weights that prompt the formulation of multivariate ratio estimators by the calibration tuning parameter subject to a pooled-calibration constraint.

2 Basic Definitions and Notations

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of size (N). Let (X) and (Y) denote the auxiliary and study variables taking values X_i and Y_i respectively on the ith unit $U_i (i = 1, 2, ..., N)$ of the population. It is assumed that $(x_i, y_i) \geq 0$, and information on the population mean $(\bar{X})$ of the auxiliary variable (X) is known. Let a sample of size (n) be drawn by simple random sampling without replacement (SRSWOR) based on which we obtain the means $(\bar{x})$ and $(\bar{y})$ for the auxiliary variable (X) and the study variable (Y).

Let the population $[U = (U_1, U_2, ..., U_N)$ of size $(N)]$ be divided into H strata with N_h units in the hth stratum from which a simple random sample of size n_h is taken without replacement. The total population size be $N = \sum_{h=1}^{H} N_h$ and the sample size $n = \sum_{h=1}^{H} n_h$, respectively. Associated with the ith element of the hth stratum are y_{hi} and x_{hi} with $x_{hi} > 0$ being the covariate; where y_{hi} is the y value of the ith element in stratum h, and x_{hi} is the x value of the ith element in stratum h, $h = 1, 2, ..., H$ and $i = 1, 2, ..., N_h$.

For the hth stratum, let $W_h = N_h/N$ be the stratum weights and $f_h = n_h/N_h$ the sample fraction.

Let the hth stratum means of the study variable Y and auxiliary variable X

$(\bar{y}_h = \sum_{i=1}^{n_h} y_{hi}/n_h$; $\bar{x}_h = \sum_{i=1}^{n_h} x_{hi}/n_h)$ be the unbiased estimator of the population mean $(\bar{Y}_h = i=1NhyhiNh ; Xh=i=1NhxhiNh$ of Y and X respectively, based on nh observations.

$S_{hx_i}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_{hi})^2$; $S_{hy}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(\bar{y}_h - \bar{Y}_h)^2$, $S_{hx_iy} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_{hi})$ $(y_h - $
$Yh, Shxixj= 1Nh- 1i=1Nhxhi- Xhi xhj- Xhj xi,st=h=1HWhxhi$ and $yst=h=1HWhyh.$

3 Review of Existing Multivariate Estimators in Sampling Survey

This section gives a review of some existing multivariate estimators in survey sampling literature that are relative to this study under the simple random sampling and the stratified random sampling designs.

3.1 Simple random sampling

3.1.1 Generalized Regression (GREG) estimator

[33], proposed the regression estimator using single auxiliary information. [34] introduced the Generalized Regression (*GREG*) Estimator while [35] studied the properties of the GREG estimator by [34] and postulated that the GREG estimators are bias-robust.

The concept of calibration estimators proposed by [20] is simply a class of linearly weighted estimators, of which the Generalized Regression (GREG) estimator is a special member. [20] have shown that all calibration estimators are asymptotically equivalent to the GREG-estimator.

The GREG-estimator under the simple random sampling design is defined as:

$$\hat{Y}_{GREG} = \hat{Y}_{HT} + \left(\sum x_k - \sum d_k x_k\right)' \hat{\mathbf{B}} \qquad (1)$$

with variance estimator given as:

$$\hat{V}(\hat{Y}_{GREG}) = \sum\sum_s (d_k d_l - d_{kl})(g_k e_k - g_l e_l) \qquad (2)$$

where

$\hat{Y}_{HT} = \sum d_k y_k$ is the Horvitz-Thompson-type estimator,

$\hat{\mathbf{B}} = (\sum d_k c_k x_k x_k')^{-1} (\sum d_k c_k x_k y_k)$ is a vector of regression coefficients obtained by fitting the regression of y on x using the data (y_k, x_k) for the element $k \in s$, $x_k = (x_{1k}, \ldots, x_{jk}, \ldots, x_{JK})'$ is a column vector, $d_k = 1/\pi_k$ is the sampling design weights where π_k is the inclusion probability $[\pi_k = \sum_{k \in s} P(s); \pi_k > 0 \ \forall \ k]$, c_k is the tuning parameter [(always specified by the Survey Statistician) for this work $c_k = 1 \ \forall \ k]$, $g_k = 1 + c_k(\sum x_k - \sum d_k x_k)'(\sum d_k c_k x_k x_k')^{-1} x_k$ is the weighting factor and $e_k = y_k - x_k' \hat{B}$ is the residual.

3.1.2   Isaki (1983) multivariate regression estimator

The [36] multivariate regression estimator in simple random sampling is given by

$$\hat{S}_{MR}^2 = S_y^2 + \sum_i B_i(S_{x_i}^2 - s_{x_i}^2) \qquad (3)$$

with variance estimator given by:

$$\hat{V}(\hat{S}_{MR}^2) = V(S_y^2) + \sum_i B_i^2 V(S_{x_i}^2) - 2\sum_i B_i Cov(S_y^2, S_{x_i}^2)$$
$$+ \sum_{i \neq j} B_i B_j Cov\left(S_{x_i}^2, S_{x_j}^2\right) \qquad (4)$$

3.2 Stratified random sampling

3.2.1 Generalized Regression (GREG) estimator

The GREG-estimator under the stratified random sampling design is defined as:

$$\hat{Y}_{st,GREG} = \hat{Y}_{st,HT} + \left(\sum x_k - \sum d_k x_k\right)' \hat{B}_{st} \tag{5}$$

with variance estimator given as:

$$\hat{V}(\hat{Y}_{st,GREG}) = \sum_{h=1}^{H} N_h^2 \, \gamma_h S_{e_{hk}}^2 \tag{6}$$

Where

$\hat{Y}_{st,HT} = \sum d_k y_{hk}$ is the Horvitz- Thompson –type estimator, $S_{e_{hk}}^2 = \frac{1}{n_h-1}\left(g_k e_k - \frac{1}{n_h}\sum g_k e_k\right)^2$ is the stratum variance of the residual e_k, $e_k = y_k - x_k'\hat{B}_{st}$, $g_k = 1 + c_k(\sum x_k - \sum d_k x_k)'(\sum d_k c_k x_k x_k')^{-1} x_k$ is the weighting factor, $\hat{B}_{st} = (\sum d_k c_k x_k x_k')^{-1} (\sum d_k c_k x_k y_{hk})$ is a vector of regression coefficients obtained by fitting the regression of y on x using the data (y_k, x_k) for the element $k \in s$, $x_k = (x_{1k}, \dots, x_{jk}, \dots, x_{JK})'$ is a column vector, $d_k = 1/\pi_k$ is the sampling design weights where π_k is the inclusion probability $[\pi_k = \sum_{k \in s} P(s); \pi_k > 0 \ \forall \ k]$, c_k is the tuning parameter.

3.2.2 Isaki (1983) multivariate regression estimator

The multivariate regression estimator in stratified random sampling established by [36] is given by

$$\hat{S}_{MR}^2 = S_{hy}^2 + \sum_{h=1}^{H} B_{hi}\left(S_{hx_i}^2 - s_{hx_i}^2\right) \tag{7}$$

with variance estimator given by:

$$\hat{V}(\hat{S}_{MR}^2) = V(S_{hy}^2) + \sum_{h=1}^{H} B_{hi}^2 \, V(S_{hx_i}^2) - 2\sum_{h=1}^{H} B_{hi} Cov(S_{hy}^2, S_{hx_i}^2)$$
$$+ \sum_{i \neq j} B_{hi} B_{hj} Cov\left(S_{hx_i}^2, S_{hx_j}^2\right) \tag{8}$$

4 The Proposed Multivariate Ratio Calibration Estimator

The calibration estimator for the stratified random sampling is defined by [37] as given by:

$$\bar{y}_{st}(Tr) = \sum_{h=1}^{H} W_h^* \bar{y}_h \tag{9}$$

where W_h^* is the calibration weights which minimizes given calibration constraint(s).

Motivated by [37], let the suggested Multivariate Ratio Calibration Estimator be defined by:

$$\hat{Y}_{MR}^* = \sum_{h=1}^{H} \phi^* \bar{y}_h \tag{10}$$

where ϕ^* is the pooled-weights called the Design- Calibration Weights.

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Let W_h^* and W_{hi} denote respectively, the calibration weights and the stratified sampling design weights associated with the ith auxiliary variable, so that the pooled-weights ϕ^* is defined by:

$$\phi^* = \sum_{i=1}^{p} W_{hi} W_h^* \tag{11}$$

where W_h^* is chosen such that the loss function $L(W_h^*, W_h)$ defined by

$$L(W_h^*, W_h) = \sum_{h=1}^{H} \frac{(W_h^* - W_h)^2}{W_h Q_{hi}} \tag{12}$$

is minimized while satisfying a pooled- calibration constraint defined by

$$\sum_{i=1}^{p} \sum_{h=1}^{H} W_h^* \bar{x}_{hi} = \sum_{i=1}^{p} X_i \tag{13}$$

where W_h is the stratum weights defined by $W_h = N_h/N$, Q_{hi} is the calibration tuning parameter, $\bar{x}_{hi}$ is the sample stratum mean of the ith auxiliary variable and X_i is the population total of the ith auxiliary variable.

Minimizing the loss function (12) subject to the pooled-calibration constraint (13) gives the calibration weights as:

$$W_h^* = W_h + \frac{W_h Q_{hi} \bar{x}_{hi}}{\sum_{i=1}^{p} \sum_{h=1}^{H} W_h Q_{hi} \bar{x}_{hi}^2} \left(\sum_{i=}^{p} X_i - \sum_{i=1}^{p} \sum_{h=1}^{H} W_h \bar{x}_{hi} \right) \tag{14}$$

Substituting (14) in (11) and the subsequent results in (10) while setting $Q_{hi} = (\bar{x}_{hi})^{-1}$ gives the proposed Multivariate Ratio Calibration Estimator of total in stratified random sampling as:

$$\hat{Y}_{MR}^* = \sum_{i=1}^{p} W_{hi} R_i X_i \tag{15}$$

$$R_i = \frac{\sum_{h=1}^{H} W_h \bar{y}_h}{\sum_{h=1}^{H} W_h \bar{x}_{hi}}$$

So that

$$\hat{Y}_{MR}^* = W_{h1} R_1 X_1 + W_{h2} R_2 X_2 + W_{h3} R_3 X_3 + \cdots + W_{hp} R_p X_p \tag{16}$$

4.1 Estimator of variance for the proposed estimator

The most challenging limitation of the ratio estimation is that of deriving variance estimator that admits more than two auxiliary variables. Many authors have proposed various forms of multivariate ratio estimators in sample surveys [see 15,17,18,38] among others] but their estimator of variance admit only two auxiliary variables (that is , bivariate ratio estimators).To derive estimator of variance that would admits as many auxiliary variables as desired, this paper adapts the variance estimation approach developed by [29] for the univariate ratio calibration estimator to multivariate ratio method estimation.

Let the multivariate ratio estimator of total under an ideal condition (without calibration) be defined by:

$$\hat{Y}_{MR} = \sum_{i=1}^{p} N\bar{y}_{R_i} \qquad (17)$$

So that

$$\hat{V}(\hat{Y}_{MR}) = \sum_{i=1}^{p} N^2 \hat{V}(\bar{y}_{R_i}) \qquad (18)$$

Similarly, let the multivariate ratio estimator of total under the calibration estimation be defined by:

$$\hat{Y}_{MR}^* = \sum_{i=1}^{p} N\bar{y}_{R_i}^* \qquad (19)$$

So that

$$\hat{V}(\hat{Y}_{MR}^*) = \sum_{i=1}^{p} N^2 \hat{V}(\bar{y}_{R_i}^*) \qquad (20)$$

The estimator of variance of the conventional combined ratio estimator by [1], in stratified sampling is given by:

$$\hat{V}(\bar{y}_{R_i}) = \sum_{h=1}^{H} w_h^2 \gamma_h S_{dh}^2 \qquad (21)$$

where $S_{dh}^2 = \left(S_{hy}^2 + R^2 S_{hx}^2 - 2RS_{hxy}\right)$

Let the estimator of variance of the proposed multivariate ratio calibration estimator under the stratified sampling be defined by:

$$\hat{V}(\bar{y}_{R_i}^*) = \sum_{h=1}^{H} W_h^{*2} \varphi_h S_{dh}^{*2} \qquad (22)$$

where φ_h is the new calibration weights, chosen such that the loss function $L(\varphi_h, \gamma_h)$ defined by

$$L(\varphi_h, \gamma_h) = \sum_{h=1}^{H} \frac{(\varphi_h - \gamma_h)^2}{\gamma_h q_{hi}} \qquad (23)$$

is minimized while satisfying the pooled-calibration constraint of the form:

$$\sum_{i=1}^{p} \sum_{h=1}^{H} \varphi_h s_{hx_i}^2 = \sum_{i=}^{p} S_{hx_i}^2 \qquad (24)$$

where $s_{hx_i}^2$ and $S_{hx_i}^2$ are known sample stratum variance and population stratum variance of the ith auxiliary variable (X_i) respectively, q_{hi} is the new calibration tuning parameter and $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$

Minimizing the loss functions (23) subject to the calibration constraint (24) gives the new calibration weights for the variance as

$$\varphi_h = \gamma_h + \frac{\gamma_h q_{hi} s_{hx_i}^2}{\sum_{i=1}^{p} \sum_{h=1}^{H} \gamma_h q_{hi} s_{hx_i}^4} \left[\sum_{i=1}^{p} S_{hx_i}^2 - \sum_{i=1}^{p} \sum_{h=1}^{H} \gamma_h s_{hx_i}^2 \right] \tag{25}$$

Substituting equation (25) in equation (22) and setting $q_{hi} = s_{hx_i}^{-2}$ gives

$$\hat{V}(\bar{y}_{R_i}^*) = \sum_{h=1}^{H} W_h^{*2} \gamma_h S_{dh}^{*2} + \frac{\sum_{h=1}^{H} W_h^{*2} \gamma_h S_{dh}^{*2}}{\sum_{i=1}^{p} \hat{S}_{hx_i}^2} \left[\sum_{i=1}^{p} S_{hx_i}^2 - \sum_{i=1}^{p} \hat{S}_{hx_i}^2 \right] \tag{26}$$

where $\hat{S}_{hx_i}^2 = \sum_{h=1}^{H} \gamma_h s_{hx_i}^2$

By setting $Q_{hi} = (\bar{x}_{hi})^{-1}$ in equation (14) and substituting the results in equations (26) and (20) respectively; gives the estimator of variance of the proposed multivariate ratio calibration estimator of total as:

$$\hat{V}(\hat{Y}_{MR}^*) = \sum_{h=1}^{H} N_h^2 \tau^2 \gamma_h \xi \, S_{dh}^{*2} \tag{27}$$

Where $S_{dh}^{*2} = \left(S_{hy}^2 + \sum_{i=1}^{p} R_i^2 S_{hx_i}^2 - 2 \sum_{i=1}^{p} R_i S_{hx_i y} + 2 \sum_{i \neq j} R_i R_j S_{hx_i x_j} \right)$,

$$\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h} \right), R_i = \sum_{h=1}^{H} w_h \bar{y}_h / \sum_{h=1}^{H} w_h \bar{x}_{hi}, \hat{\bar{X}}_i = \sum_{h=1}^{H} w_h \bar{x}_{hi}$$

$\tau = \left(\frac{\sum_{i=}^{p} X_i}{\sum_{i=1}^{p} \hat{X}_i} \right), \xi = \left(\frac{\sum_{i=}^{p} S_{hx_i}^2}{\sum_{i=1}^{p} \hat{S}_{hx_i}^2} \right)$, $W_h [= N_h/N]$ is the stratum weights and $S_{hx_i x_j}$ is the population stratum covariance between the ith auxiliary variable and the jth auxiliary variable, $S_{hx_i y}$ is the population stratum covariance between the ith auxiliary variable and the study variable and S_{hy}^2 is the population stratum variance of the study variable.

It should be noted here that τ and ξ are the precision factors attributed to the pooled-constraints of equations (9) and (20) respectively.

Under an ideal condition the estimator of variance of the multivariate ratio estimator of total in stratified random sampling is given by:

$$\hat{V}(\hat{Y}_{MR}) = \sum_{h=1}^{H} N_h^2 \gamma_h \left[S_{hy}^2 + \sum_{i=1}^{p} R_i^2 S_{hx_i}^2 - 2 \sum_{i=1}^{p} R_i S_{hx_i y} + 2 \sum_{i \neq j} R_i R_j S_{hx_i x_j} \right] \tag{28}$$

Under the calibration estimation, the estimator of variance of the multivariate ratio estimator of total in simple random sampling without replacement (SRSWOR) is given by:

$$\hat{V}(\hat{Y}_{MR}^*) = N^2 \tau^2 \xi \gamma \left(S_y^2 + \sum_{i=1}^{p} R_i^2 S_{x_i}^2 - 2 \sum_{i=1}^{p} R_i S_{x_i y} + 2 \sum_{i \neq j} R_i R_j S_{x_i x_j} \right) \tag{29}$$

Where,

$\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$, $R_i = \bar{y}/\bar{x}_i$, $\hat{X}_i = \sum_{i=1}^{n} \bar{x}_i$ $\tau = \left(\frac{\sum_{i=}^{p} x_i}{\sum_{i=1}^{p} \hat{X}_i}\right), \xi = \left(\frac{\sum_{i=}^{p} S_{x_i}^2}{\sum_{i=1}^{p} \hat{S}_{hx_i}^2}\right)$, $S_{x_i x_j}$ is the population covariance between the ith auxiliary variable and the jth auxiliary variable, $S_{x_i y}$ is the population covariance between the ith auxiliary variable and the study variable and S_y^2 is the population variance of the study variable.

Similarly, under an ideal condition the estimator of variance of the multivariate ratio estimator of total in simple random sampling without replacement (SRSWOR) is given by:

$$\hat{V}(\hat{Y}_{MR}) = N^2 \gamma [S_y^2 + \sum_{i=1}^{p} R_i^2 S_{x_i}^2 - 2 \sum_{i=1}^{p} R_i S_{x_i y} + 2 \sum_{i \neq j} R_i R_j S_{x_i x_j}] \tag{30}$$

5 Empirical Study

An empirical study was carried out to estimate the population total of a simulated population and compare the performance of the proposed estimator to that of GREG-estimator.

5.1 Background and analytical set-up

A population $(y_h \ and \ x_{hi} \ i = 1,2,3,4)$, which has 8 strata in which each stratum differs from others was simulated. The difference was achieved by using different error terms e_h. An assisting model of the form: $y_h = \beta_0 + \beta_1 x_{1h} + \beta_2 x_{2h} + \beta_3 x_{3h} + \beta_4 x_{4h} + e_h$ was designed to generate the y_h where h is the number of strata $(h = 1,2,...,8)$ and e_h are independently generated by the standard normal distribution. The coefficients β_i $(i = 0,1,2,3,4)$ were randomly generated from a uniform distribution while $y_h, x_{1h}, x_{2h}, x_{3h}$ and x_{4h} were randomly generated from normal distribution with different parameters.

The simulation study was conducted using the R-statistical package. There were $M = 1,500$ for the m-th run $(m = 1,2,...,M)$, a Bernoulli sample is drawn where each unit is selected into the sample independently, with inclusion probability $\pi_i = n/N$. For simplicity the tuning parameter Q_{hi} was set to unity $(Q_{hi} = 1)$. A sample of size 200 was selected randomly from the simulated population index-wise, that is if index h is selected then the sample elements will have $y_h, x_{1h}, x_{2h}, x_{3h}$ and x_{4h}. The corresponding GREG-estimator and calibration estimator of Y were computed. The results of the analysis are given in Tables 1 and 2.

5.2 Comparisons with existing estimators

For a given estimator (say) $\hat{Y}_i^*$, let $\hat{Y}_i^{*(m)}$ be the estimate of $\hat{Y}_i^*$ in the *m-th* simulation run; $m =$1, 2... M (=1,500). To compare the performances of the proposed multivariate calibration ratio estimator with that of the GREG-estimator the following criteria; bias (B), mean square error (*MSE*) and the percentage relative efficiency (*PRE*) were used. Each measure is calculated as follows:

(i) $B\left(\overline{\hat{Y}}_i^*\right) = \overline{\hat{Y}}_i^* - \widehat{\overline{Y}}_i^{*(m)}$

where $\overline{\hat{Y}}_i^* = \frac{1}{M} \sum_{m=1}^{M} \hat{Y}_i^{*(m)}$

(ii) $MSE\left(\hat{Y}_i^*\right) = \sum_{m=1}^{M} \left(\hat{Y}_i^{*(m)} - \overline{\hat{Y}}_i^*\right)^2 / M$

where $\hat{Y}_i^{*(m)}$ is the estimated total based on sample m and M is the total number of samples drawn for the simulation.

(iii) The percent relative efficiency (PRE)

The percent relative efficiency (PRE) of an estimator θ with respect to the [36] multivariate regression estimator $\left(\hat{S}_{MR}^2\right)$ in stratified sampling is defined by:

$$PRE\left(\theta, \hat{S}_{MR}^2\right) = \frac{V\left(\hat{S}_{MR}^2\right)}{V(\theta)} \times 100$$

Table 1. Performance of estimators

Estimator	Simple random sampling			Stratified random sampling		
	Bias	MSE	PRE	Bias	MSE	PRE
$\hat{S}_{MR}^2$	4.368	3648.864	100.00	2.482	2274.372	100.00
$\hat{Y}_{GREG}$	2.484	1983.026	184.005	1.301	1263.148	180.056
$\hat{Y}_{MR}^*$	0.623	1402.234	260.022	0.482	893.243	254.620

Table 2. MSEs of proposed estimators under different conditions

Estimator	Ideal condition	Calibration design
$\hat{Y}_{MR}^*$	1872.0492	1402.234
$\hat{Y}_{st,MR}^*$	1178.684	893.643

6 Discussion of Results

Table 1 summarizes the statistics corresponding to each estimator under simple random sampling and stratified random sampling. Table 2 gives the MSEs of the proposed estimator under ideal condition and calibration design.

The true population (simulated) total is13, 565 while the estimated total by the GREG and proposed estimators are 13,654.045 and 13,564.632 respectively. It is evident that the proposed multivariate ratio calibration estimator is a better approximation of the true population total than the GREG-estimator.

Numerical results for the Percent Relative Efficiency (*PREs*) under the simple random sampling reveals that the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}_{MR}^*\right)$ has 160 percent gains in efficiency while the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ has 84 percent gains in efficiency; this shows that the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}_{MR}^*\right)$ is 76 percent more efficient than the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$.

Similarly, numerical results for the Percent Relative Efficiency (*PREs*) under the stratified random sampling reveals that the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}_{MR}^*\right)$ has 155 percent gains in efficiency while the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ has 80 percent gains in efficiency; this shows that the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}_{MR}^*\right)$ is 75 percent more efficient than the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$.

This means that in using the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}_{MR}^*\right)$ one will have 76 and 75 percent efficiency gain over the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ under the simple random sampling and stratified random sampling respectively.

Table 2 showed that the proposed estimators under the calibration design are respectively more efficient than their corresponding counterparts under ideal condition both under the simple random sampling and stratified

random sampling. This is in line with established fact in the literature of sampling survey that estimation under the calibration design gives better appealing results than estimation under ideal condition [39-40].

In terms of bias, the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}^*_{New}\right)$ is better than the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ under all sampling designs considered.

7 Conclusion

Sequel to the discussion of results above, it is concluded that the proposed Multivariate Ratio Calibration Estimator $\left(\hat{Y}^*_{MR}\right)$ fares better than the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ both in efficiency and biasedness. This is against an established fact in survey sampling literature that the Generalized Regression Estimator $\left(\hat{Y}_{GREG}\right)$ is always more efficient than both the ratio and product estimators.

Therefore, the suggested Multivariate Ratio Calibration Estimator is very attractive to survey researchers as it gives consistent and more precise estimates of the population parameters.

Competing Interests

Author has declared that no competing interests exist.

References

[1]   Hansen MH, Hurwitz WN, Gurney M. The problems and methods of the sample survey of business. Journal of the American Statistical Association. 1946;41:173-189.

[2]   Singh GN. On the improvement of product method of estimation in sample surveys. Journal of Indian Society of Agricultural Statistics. 2003;56(3):267-275.

[3]   Singh HP, Vishwakarma GK. Modified exponential ratio and product estimators for finite population mean in double sampling. Austrian Journal of Statistics. 2007;36(3):217-225.

[4]   Singh R, Chauhan, P, Sawan N, Smarandache F. Improvement in estimating the population mean using exponential estimator in simple random sampling. Bulletin of Statistics and Economics. 2009; 3(9):13-18.

[5]   Sharma B, Tailor R. A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. Global Journal of Science. 2010;10(1):27-31.

[6]   Choudhury S, Singh BK. A class of chain ratio-cum-dual to ratio type estimator with two auxiliary characters under double sampling in sample surveys. Statistics in Transition-New Series. 2012;13(3): 519-536.

[7]   Khare BB, Sinha RR. Combined class of estimators for ratio and product of two population means in presence of non-response. International Journal of Statistics and Economics. 2012;8(12):12-20.

[8]   Khare BB, Srivastava U, Kumar K. Generalized chain type estimators for ratio of two population means using two auxiliary characters in the presence of non-response. International Journal of Statistics and Economics. 2013;10(1):51-64.

[9]   Singh R, Audu A. Efficiency of ratio estimator in stratified random sampling using information on auxiliary attribute. International Journal of Engineering and Innovative Technology. 2013;2(1):116-172.

[10] Clement EP. An improved ratio estimator for population mean in stratified random sampling. European Journal of Statistics and Probability. 2016;4(4):12-17.

[11] Etuk SI, Enang EI, Ekpenyong EJ. A modified class of ratio estimators for population mean, Communications in Statistics -Theory and Methods. 2016;45(4):837-849.
DOI: 10.1080/03610926.2013.851238

[12] Clement EP. Efficient exponential estimators of population mean in survey sampling. International Journal of Mathematics and Computation. 2017;28(3):94-106.

[13] Clement EP. Generalized chain ratio-product estimator for estimating population mean with Auxiliary variate. Elixir Statistics. 2017;106:46471-46478.

[14] Olkin I. Multivariate ratio estimation for finite population. Biometrica. 1958;45:154-165.

[15] Singh MP. Multivariate product method of estimation for finite populations. Journal of the Indian Society of Agricultural Statistics. 1967;19:1–10.

[16] Singh R, Chauhan P, Sawan N, Smarandache F. Auxiliary information and a priori values in construction of improved estimators. Renaissance High Press, Ann Arbor, Mich, USA; 2007.

[17] Malik S, Singh R. Some improved multivariate-ratio-type estimators using geometric and harmonic means in stratified random sampling international scholarly research network: Probability and Statistics; 2012.
DOI: 10.5402/2012/509186

[18] Ngesa OO, Orwa GO, Otieno RO, Murray HM. Multivariate ratio estimator of the population total under stratified random sampling. Open Journal of Statistics. 2012;2:300-304.
Available:dx.doi.org/10.4236/ojs.2012.23036

[19] Clement EP, Enang EI. Multivariate calibration estimation for domain in stratified random sampling. International Journal of Modern Mathematical Sciences. 2015;13(2):187-197.

[20] Deville JC, Sarndal CE. Calibration estimators in survey sampling. Journal of the American Statistical Association. 1992;87:376-382.

[21] Arnab R, Singh S. A note on variance estimation for the generalized regression predictor. Australia and New Zealand Journal of Statistics. 2005;47(2):231-234.

[22] Estavao VM, Sarndal CE. Survey estimates by calibration on complex auxiliary information. International Statistical Review. 2006;74:127-147.

[23] Kott PS. Using calibration weighting to adjust for non-response and coverage errors. Survey Methodology. 2006;32:133-142.

[24] Sarndal CE. The calibration approach in survey theory and practice. Survey Methodology. 2007;33: 99-119.

[25] Clement EP. Calibration approach separate ratio estimator for population mean in stratified sampling. International Journal of Modern Mathematical Sciences. 2015;13 (4):377-384.

[26] Clement EP. A new ratio estimator of mean in survey sampling by calibration estimation elixir statistics. 2017;106:46461-46465.

[27] Clement EP, Udofia GA, Enang EI. Estimation for domains in stratified random sampling in the presence of non-response. Aligarch Journal of Statistics. 2015;35:117-132.

[28] Clement EP, Enang EI. Calibration approach alternative ratio estimator for population mean in stratified sampling. International Journal of Statistics and Economics. 2015;16(1):83-93.

[29] Clement EP, Enang EI. On the efficiency of ratio estimator over the regression estimator. Communication in Statistics: Theory and Methods. 2017;46(11):5357-5367.
DOI: 10.1080/03610926.2015.1100741

[30] Kim JK, Park M. Calibration estimation in survey sampling. International Statistical Review. 2010; 78(1):21-29.

[31] Clement EP. Estimation of population mean in calibration ratio-type estimator under systematic sampling elixir statistics. 2017;106:46480-46486.

[32] Enang EI, Clement EP. An efficient class of calibration ratio estimators of domain mean in survey sampling. Communication in Mathematics and Statistics; 2019.
DOI: 10.1007/s40304-018-00174-z

[33] Cochran WG. Sampling theory when the sampling units are of unequal sizes. Journal of the American Statistical Association. 1942;37:199-212.

[34] Sarndal CE, Swenson B, Wretman JH. Model-assisted survey sampling. New York: Springer-Verlag; 1992.

[35] Beaumont JF, Alavi A. Robust generalized regression estimation. Survey Methodology. 2004;30:195-208.

[36] Isaki CT. Variance estimation using auxiliary information. Journal of the American Statistical Association. 1983;78:117-123.

[37] Tracy DS, Singh S, Arnab R. Note on calibration in stratified and double sampling. Survey Methodology. 2003;29:99-104.

[38] Singh MP. On the estimation of ratio and product of the population parameters. Sankhya. 1965;27: 321–328.

[39] Kim JM, Sungur EA, Heo TY. Calibration approach estimators in stratified sampling. Statistics and Probability Letters. 2007;77(1):99-103.

[40] Rao D, Khan MGM, Khan S. Mathematical programming on multivariate calibration estimation in stratified sampling, World Academy of Science, Engineering and Technology. 2012;72:12-27.

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